Elementary Applied Symbolic Logic

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STATEMENTS; TRUTH-VALUES; COMPOUNDS; ABBREVIATIONS

§1.1 Statements

The basic concept in logic is that of the **statement** or **proposition**. (These two terms are used interchangeably throughout the book.) A definition adequate for our purposes is the following:

> *Statement (Proposition).* A statement (or proposition) is what is expressed in uttering—vocally, or in writing, or in some other way—a declarative sentence on a particular occasion of normal discourse.*

While declarative sentences are the vehicles of statements, a statement is not the same thing as a sentence. Statements are abstractions, which cannot be equated with the sentences used to express them for two obvious reasons: First, any given sentence may be used, on different occasions, to make *different* statements. Second, different sentences may sometimes be used to make the *same* statement. For example, the sentence

1. John is bald.

uttered by someone in Detroit to say something about his brother, would on that occasion express one proposition, while the same sentence uttered by someone in Los Angeles to say something about his parrot would express a different proposition. Similarly, the two different sentences

2. Today is Wednesday.
3. Yesterday was Wednesday.

might be used to express the same proposition; for example, if (2) were uttered one day and (3) were uttered the next.

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*See Appendix A for more on this subject. The appendices contain supplementary information for the interested reader. They need not be consulted to understand the text.

Sentences are identified and distinguished by the words they contain and their grammatical form. *Propositions* are identified and distinguished by their truth-conditions—that is, the conditions that would have to obtain in order for them to be true or to be false. Different truth-conditions indicate different propositions; and, for our purposes, identical truth-conditions indicate identical propositions. For example, returning to (1), let us suppose that the man in Detroit is named Jones and the man in Los Angeles is named Smith. Then, in order for Jones' statement to be true, Jones' brother would have to be bald; and in order for Smith's statement to be true, Smith's parrot would have to be bald. But the condition of Jones' brother being bald, and the condition of Smith's parrot being bald, are quite different. Hence the two statements are different statements. On the other hand, if (2) and (3) are uttered on consecutive days, so that the pronouns 'today' and 'yesterday' come to designate the same day, then both statements will be true if that day happens to be Wednesday and both will be false if it happens not to be Wednesday. The truth-conditions for the two statements are identical; hence, we can say the two statements are identical. In general, any pair of natural-language statements may be treated as identical if, and only if, they have the same truth-conditions.

Not every utterance of a declarative sentence expresses a proposition. Only those which are genuine portions of discourse do so. The sentence "John is bald" uttered above does not express a proposition; it was uttered simply as an example of a sentence, not as an attempt to communicate information to you about anyone named John. Most of the example-sentences in any logic book will fail to express propositions for the same reason. However, since it is virtually impossible to teach or learn logic without the help of examples, we shall treat our examples as if they were genuine portions of discourse, uttered by someone for the purpose of communication. Henceforth, every sentence employed as an example will (unless otherwise indicated) be treated as if it expressed a proposition.

Also, for purposes of simplicity and to avoid confusion, we shall stipulate two further conventions about sentences and statements, which will be adhered to throughout the remainder of the book.

"SAME SENTENCE SAME STATEMENT" CONVENTION. Unless otherwise stated, repeated occurrences of the same sentence, within a given context, will be regarded as repetitions of the same statement. (This means that we shall not allow a sentence to express different propositions in the same context.)

"SAME NAME SAME REFERENCE" CONVENTION. Unless otherwise stated, repeated occurrences of the same name within a given context will be regarded as references to the same thing. (This means, for example, that John Smith and John Jones cannot both be referred to simply as 'John' in the same context.)

§1.2   Truth-Values

Statements can be either true or false; that is, they have **truth-values**. There are two truth-values: True and False. A true statement has a truth-value of True; a false statement has a truth-value of False. There is no third truth-value in addition to these two.*

No statement can have both truth-values. No statement is both true and false. So far as truth-values go, a miss is as good as a mile. A baseball player who was "almost out" is safe, and one who was "almost safe" is out. Similarly, a proposition that is "almost true" is nevertheless false, and one that is "almost false" is nevertheless true.

So far as we are concerned, every statement has at least one of the two truth-values. It is a matter of some philosophical dispute whether or not all statements satisfy this condition: it has been argued that some statements (for example, those having to do with moral or religious matters) are neither true nor false. This dispute is irrelevant to elementary logic. If there are any statements lacking a truth-value, they fall entirely outside the scope of our discipline and are of no interest to us. So far as the *logic* of statements is concerned, it is regarded as an axiom that every statement up for logical consideration is either true or false.†

From the fact that every statement has a truth-value and no statement has both, it follows that every statement has just one of the two truth-values. A statement which is not true is false, and one which is not false is true. Likewise, a statement which is false is not true and one which is true is not false.

§1.3   Simple and Compound Statements

Any declarative sentence which contains another declarative sentence as an independent clause is referred to in logic as a **compound sentence**. The notion of an "independent" clause, as given in most grammar books, is rather vague; but for present purposes we may take it to mean a clause (that is, a grammatical unit of the sentence) whose truth-conditions could be specified in ignorance of the remainder of the sentence in which the clause occurs. For logical purposes, "function words" (prepositions, conjunctions, conjunctive adverbs, relative pronouns, and so on) preceding or following a clause are not regarded as part of the clause itself. This notion can best be explained

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*Professional logicians sometimes study what are called "multiple-valued logics." But the values involved in addition to truth and falsity are not *truth*-values.
†This is sometimes called the "Law of Excluded Middle": There is no "middle ground" between truth and falsity.

with the help of examples. Consider

1. The boy who kicked Evangeline is a brat.

Even though the last four words of this sentence, 'Evangeline is a brat', have the grammatical form of a declarative sentence, it would be a mistake to regard (1) as a compound sentence on these grounds, since the last four words do not constitute a *clause* within the sentence; rather, 'Evangeline' is a portion of the subject-phrase, and 'is a brat' is the predicate of the whole sentence. In sorting out the independent clauses within sentences it is often helpful to enclose them in parentheses. But if we attempt to analyze (1) as

1a. The boy who kicked (Evangeline is a brat).

the result is not even a *sentence*; (1a) does not express anything with truth-conditions. If (1) is analyzed as (1a) it does not make a coherent assertion, which is sufficient to indicate that (1a) is the wrong analysis. We can state this by saying that the sentence 'Evangeline is a brat' is not a *component* of sentence (1), since it is not an independent clause within that sentence. As another example, consider

2. It is true that turtles lay eggs.

which contains two declarative sentences as clauses:

2a. (It is true) that (turtles lay eggs).

However, only the second of these is an *independent* clause. 'It is true' is not an independent clause, since one could not specify its truth-conditions without knowing *what it is* that is supposed to be true; that is, the remainder of the sentence. On the other hand, 'turtles lay eggs' expresses a proposition with independently specifiable truth-conditions, and so is an independent clause which is a *component* of sentence (2). Similarly for

3. Clyde revealed that Harvey is a policeman.

Although this may be analyzed as

3a. (Clyde revealed) that (Harvey is a policeman).

the first clause in (3a) is not an independent clause. 'Reveal' is a transitive verb, and there is no way to describe the truth-conditions of a statement involving this verb without knowing its direct object, which in the present case is the proposition that Harvey is a policeman—that is, the remainder of the sentence. Thus (3) is a compound sentence with only one component: the clause 'Harvey is a policeman'. As another example, in

4. Jones was fired by the company for embezzlement.

the clause 'Jones was fired by the company' is an independent clause and thus is a component of (4). (Or, if we take the verb to be the intransitive 'fired', rather than the transitive 'fired by', the clause 'Jones was fired' may also be taken as an independent clause.) Although the adverbial phrase 'for embez-

zlement' modifies something within the independent clause, it is not *part* of the independent clause, since it is not part of the declarative sentence constituting that clause.

Clauses may be regarded as independent even though there is a pronominal overlap between them. For example, the sentence

5. Evangeline walked in and Percy kicked her.

should be regarded as containing two independent clauses 'Evangeline walked in' and 'Percy kicked her', even though the personal pronoun in the second clause has its antecedent in the first clause. A referential overlap of this sort does not destroy the independence of the two clauses. This is quite different from cases such as (2) and (3), in which we literally cannot understand what the dependent clause asserts until we understand (not merely the reference, but) the entire content of the independent clause. Thus (5) is a compound sentence with two components, as is

6. Either it will rain tonight or the weatherman is incompetent.

in which the two components 'it will rain tonight' and 'the weatherman is incompetent' are totally independent.

An independent clause in a compound sentence may itself be a compound sentence. For example, in the sentence

7. If it rained last night and the park is all wet then the picnic has been cancelled.

the independent clauses can be presented as

7a. If (it rained last night and the park is all wet) then (the picnic has been cancelled).

However, the first of these is itself a compound sentence whose component clauses are 'it rained last night' and 'the park is all wet'; thus, the full analysis of (7) will be

7b. If ((it rained last night) and (the park is all wet)) then (the picnic has been cancelled).

The statements expressed by compound sentences are called **compound statements**. A statement which is not compound is called a **simple statement**. Simple statements are expressed by sentences which are not compound.* Just as the independent clauses of a compound sentence are called its **components**, the propositions expressed by them are called the **components** of the compound statement expressed by the whole sentence.

In logic, **negative** statements are always regarded as compound—that is, as the denials of their affirmative counterparts. For example, the negative statement

8. It won't rain tonight.

*Cf. Appendix B.

is to be treated as if it said

8a. It's not the case that it will rain tonight.

which is a compound whose component is the statement 'It will rain tonight'. This treatment is invariant and is employed even when (as in this illustration) the sentence expressing the negative statement is not phrased as a compound sentence.

Exercises I

Decide which of the following statements are simple and which are compound. For the compound ones, decide what their components are.

*□ 1. They invited me to the party but I didn't go.

2. If Harvey drank from that bottle he is going to die.

□ 3. Today is Wednesday.

4. Clyde believes that the earth is flat.

□ 5. My car is better than yours.

6. Thelma told Harvey that she had a headache.

□ 7. The earth is not flat.

8. Clyde pinched Edna for laughing too loud.

□ 9. Anyone who can swim the Atlantic Ocean is a very athletic person.

10. I won't go unless you beg me.

There are two different sorts of compound statements. On the one hand, some compound statements are such that the truth-values of their components make no difference at all to the truth-value of the whole compound. An example is

9. Harvey says that there is life on Venus.

This is a compound whose component is the statement: 'There is life on Venus'. Now whether or not there is life on Venus is immaterial to the truth (or falsity) of (9). The only thing relevant to its truth-value is whether or not Harvey has said what is attributed to him.

On the other hand, some compound statements are such that the truth-values of their components *do* make a difference to the truth-value of the whole compound. Compound statements of this sort are called **truth-dependent compounds**. An example would be

10. Harvey hit Edna in order to make her shut up.

This is a compound whose component is 'Harvey hit Edna'. And if this component happened to be false (that is, if Harvey didn't hit Edna at all),

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*An open square indicates that the answer to the problem may be found in the back of the book.

then the whole compound would also be false. Of course, if the component happened to be *true*, the whole compound might still be false (he might have hit her for some other reason); but nevertheless the truth-value of the component is, to a certain extent, relevant to the truth-value of the whole. Another example of a truth-dependent compound is

11. Clyde drinks a lot but Harvey is a teetotaler.

This is a compound whose components are 'Clyde drinks a lot' and 'Harvey is a teetotaler'. It is truth-dependent, since if either one of these (or both of them) happened to be false, then the whole compound would also be false. However, if both components happen to be *true*, then the whole compound will also be true. This means that the truth-value of (11) is *completely* determined by the truth-values of its components: if both of its components are true, then it is true; otherwise, it is false. A compound statement of this sort is said to be **truth-functional**, since its truth-value is a function of the truth-values of its components.

> *Truth-Function.* X is a truth-function of Y if and only if the truth-value(s) of Y completely determine(s) the truth-value of X. Thus every statement is, trivially, a truth-function of itself.

> *Truth-Functional Compound.* A compound statement is a truth-functional compound if and only if its truth-value is completely determined by the truth-values of its components. In such a circumstance, the compound statement is also said to be a truth-function of its components.

It should be obvious that every *negative* statement is a truth-functional compound, since the negative statement will be true if its component is false, and false if its component is true.

The first area of logic to be studied is called **Truth-Functional Logic** (or **Propositional Logic**). It is concerned exclusively with truth-functional structures and the logical relationships that obtain between statements in virtue of their truth-functional makeup. Thus, the only compound statements that will be of logical interest will be the truth-functional compounds.* For this reason, in truth-functional logic all statements that are not truth-functional compounds are lumped together as "simple statements." A statement which is not a truth-functional compound is, from a logical point of view, simple. From now on we shall not bother to differentiate between *bona fide* simple statements and "simple" statements which are really some sort of non-truth-functional compound, and we shall use the word "compound" as a synonym for "truth-functional compound," unless there is a specific indication to the contrary.

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*Appendix C gives procedures for treating certain non-truth-functional compounds.

§1.4 Abbreviating Simple Statements

In truth-functional logic no attention is paid to the internal structure or subject matter of simple statements. They are of interest only insofar as they can be spliced together into different kinds of truth-functional compounds. For this reason, it is standard practice to abbreviate simple statements as drastically as possible. This is done by abbreviating them down to a single *capital* letter—generally the initial letter of some key word in the statement, though this is not mandatory. One of the first preliminaries to all logical operations is to abbreviate the simple statements (including simple components of truth-functional compounds) being considered. For example, if we wish to "prepare"

    1. Thelma lives in the country but Harvey lives in town.

for logical treatment, we first identify its simple components. These are

    1a. Thelma lives in the country.
    1b. Harvey lives in town.

We then abbreviate these. For example, (1a) might be abbreviated as '*C*', and (1b) might be abbreviated as '*T*'. Applying these abbreviations to (1) we obtain the drastically shortened statement

    1c. *C* but *T*.

Occasionally, compound statements contain "hidden components," or components which are not explicitly presented by means of a fully articulated sentence. Before performing the abbreviations in such cases it is helpful to paraphrase the compound statement into a form which explicitly presents all components. For example, the statement

    2. Clyde was injured in the accident even though Harvey wasn't.

has, as its second component, the statement 'Harvey wasn't injured in the accident', though this statement isn't presented by means of a full sentence. Thus, it will be helpful to paraphrase (2) as

    2a. Clyde was injured in the accident even though Harvey wasn't injured in the accident.

The second component is negative and therefore compound. It is helpful to paraphrase negative statements to bring out explicitly their affirmative component. One way of doing this is to paraphrase 'Harvey wasn't injured in the accident' as 'It's not the case that Harvey was injured in the accident'. Following this procedure, we would paraphrase (2a) as

    2b. Clyde was injured in the accident even though it's not the case that Harvey was injured in the accident.

The simple components of (2b) are the two statements 'Clyde was injured in the accident' and 'Harvey was injured in the accident'. Abbreviating these,

respectively, as '*C*' and '*H*', we obtain the full abbreviation of (2b):

    2c. *C* even though it's not the case that *H*.

Similarly, the statement

    3. We have no milk, but plenty of eggs.

might be successively paraphrased and abbreviated as

    3a. We have no milk, but we have plenty of eggs.
    3b. It's not the case that we have milk, but we have plenty of eggs.
    3c. It's not the case that *M*, but *E*.

When abbreviating simple statements, there is one rule we *must* adhere to. It is an instance of the general rule of consistency: "Never try to make one symbol or expression do two jobs at once." Here, it may be stated as:

> CONSISTENCY: NEVER use the same capital letter to abbreviate two *different* statements in the same context. Always choose a different capital letter for each different statement.

Exercises II

Abbreviate the following statements, using paraphrase as necessary.

☐ **1.** I didn't go to the party, even though I was invited.

    2. Clyde will cry if his new paint job is scratched.

☐ **3.** Harvey has gone to either Canada or Sweden.

    4. It's false that I can't swim.

☐ **5.** If Clyde wore his boots in, then he will drown whether or not he knows how to swim.

    6. We sell cars and trucks.

☐ **7.** If it's raining or snowing we won't go on the picnic unless the weather improves.

    8. Clyde and Thelma both moved to Argentina.

chapter 2

TRUTH-FUNCTIONAL OPERATORS;
CONJUNCTION AND NEGATION;
TRUTH-TABLE DEFINITIONS;
PUNCTUATION

§2.1 Truth-Functional Operators

A **truth-functional operator** is an expression or symbol (i.e., a piece of language) which, when suitably attached to a statement or pair of statements, produces a truth-functional compound of the statements to which the operator was attached.* English contains a great many truth-functional operators, some of which we have already looked at briefly. For example, the common word 'not' is a truth-functional operator which, suitably placed within a sentence, produces a statement which is a truth-function of the original statement. Likewise, the word 'but' is a truth-functional operator which, when located between a pair of sentences, produces a compound which is a truth-function of the original statements.

Every truth-functional compound consists of a collection of simple components operated upon in various ways by truth-functional operators. As has already been noted, a truth-functional compound may have components which are themselves truth-functional compounds. But no matter how complex a truth-functional compound is, it ultimately consists of simple components hinged together by truth-functional operators.

The artificial symbolic language of truth-functional logic has a standing vocabulary which consists almost entirely of truth-functional operators. There are five such operators: '~' (*tilde*), '·' (*dot*), '∨' (*wedge*), '⊃' (*horseshoe*), and '≡' (*triple bar*). Each corresponds (in a manner to be explained) to a number of English truth-functional operators. "Logical translation" consists in translating English operators into corresponding operators from the symbolic language. In order to do this accurately, it is necessary to understand what the symbolic operators mean and what it is for an English opera-

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*Truth-functional operators are also called "truth-functional connectives," which tends to be confusing in the case of such operators as 'not', which don't serve to connect anything to anything. Suitably understood, however, either terminology is acceptable.

tor to correspond to a symbolic one. These matters are taken up in this and succeeding chapters.

§2.2 Conjunction

When the operator-word 'but' is used to connect two statements together, the resulting compound will be true if both of its components are true, and false otherwise (that is, false if either or both of the components are false). For example, the statement

1. Harvey lives in Paraguay but his wife lives in Brazil.

is false unless *both* of its components are true; the whole compound is true only if it is true that Harvey lives in Paraguay and it is also true that his wife lives in Brazil.

A compound which is true if both its components are true and false otherwise is called a "truth-functional conjunction," or more simply a **conjunction**. Thus, (1) above is a conjunction. An operator which produces a conjunction is a **conjunctive operator**. Other conjunctive operators in English are 'and', 'however', and 'although', to mention a few. The components of a conjunction are called its **conjuncts**.

In logic there is only one conjunctive operator: the symbol '·', called **dot**. Whenever the dot is placed between two statements, abbreviated or otherwise, the result is a compound statement, a conjunction, whose components are the statements that the dot occurs between. The dot has no exact synonym in English; it is a *pure* truth-functional operator, which is to say that it has no meaning beyond its ability to hook statements together into conjunctions. Nevertheless, the dot corresponds to a great many expressions in English; namely, those expressions which are conjunctive operators.

The two English conjunctive operators 'and' and 'however' do not mean the same thing; there are situations in normal discourse when it would be proper to use the one and quite inappropriate to use the other. But logic is not concerned with subtle nuances of meaning; hence, when performing logical translation it is the standard practice to replace (translate) *all* conjunctive operators with the dot, effectively ignoring their differences of meaning and concentrating only upon the truth-conditions of the compounds formed with their assistance.

A logical translation of (1), then, would proceed as follows. First its simple components are isolated and abbreviated; perhaps 'Harvey lives in Paraguay' is abbreviated as '*P*' and 'his wife lives in Brazil' is abbreviated as '*B*', resulting in

1a. *P* but *B*

The conjunctive operator 'but' is then translated as the dot, resulting in

   1b. $P \cdot B$

which is the completed logical translation of (1). As another example, take the statement

   2. Although Harvey is quite ugly, he's not very intelligent.

This paraphrases as

   2a. Although Harvey is quite ugly, it's not the case that he's very intelligent.

which abbreviates to

   2b. Although U, it's not the case that I.

Finally, translating the conjunctive operator 'although' by the dot, we get

   2c. $U \cdot$ it's not the case that I

The dot is a "word" in the symbolic language of formal logic. If a precise definition must be given, we can say:

> *The dot means that the compound formed by inserting it between two statements is true when both components are true, and false if either or both components are false.*

§2.3 Truth-Table Definitions

There is a more convenient way, however, to give the definition of the dot. This is by means of a diagram called a **truth-table**. Briefly stated, a truth-table is a diagram for displaying all possible combinations of truth-values of a set of statements and for showing how each such combination will affect the truth-value of a compound having those statements as components. The above "definition" of the dot says, in effect, that if p is a statement and q is a statement, then the compound statement $p \cdot q$ is *true* if p is true and q is true; it is *false* if p is true and q is false; it is *false* if p is false and q is true; and it is *false* if p is false and q is false. All this is represented quickly and conveniently in the "truth-table definition" of the dot:

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

In this diagram the two columns to the left (called the **base columns**) display the four possible combinations of truth-values that a pair of statements can have: they may both be true (top row), or the first may be true and the second

false (second row down), or the first may be false and the second true (third row down), or both may be false (bottom row). The column to the right shows how each of these possibilities will affect a compound formed by inserting the dot between the two statements—that is, that the compound will be true when both components are true, and false otherwise. Reading across the diagram one row at a time, we see that when the components are both True, the compound is True; when the first component is True and the second is False, the compound is False; and so on. Because the dot is a pure truth-functional operator, a truth-table definition is a *complete* definition: the information in the diagram completely exhausts the meaning of the dot.

When reading the symbolic language aloud, it is conventional to read the dot either as "dot" or as "and." However, it should be borne in mind that it is not intended to be a synonym for the English word "and," and that so reading it is merely a matter of convenience.

TRANSLATION AID

The following is a list of some of the more common English conjunctive operators:

and	not only . . . but also . . .
but	in spite of the fact that
although	but even so
however	plus the fact that
whereas	inasmuch as (*but not* 'insofar as')
also	while (*in the sense of* 'although', *not in the sense of* 'during which time')
besides	
both . . . and . . .	since (*in the sense of* 'whereas', *not in the sense of* 'after')
nevertheless	
even though (*but not* 'even if')	as (*in the sense of* 'whereas')

§2.4 Negation

When the operator-word 'not' is appropriately inserted into a sentence, the result is a compound statement that will be true if its component is false, and false if its component is true. For example, the statement

   1. Harvey is not married.

will be true if the component statement 'Harvey is married' is false; but it will be false if the component 'Harvey is married' is true. Its truth-value is always the opposite of its component's truth-value.

A compound which is true if its component is false and false if its component is true is called a **negation**. The component of a negation is called its

negate. Thus, (1) is a negation, whose negate is the statement 'Harvey is married'. An operator which produces a negation when applied to a statement is called a **negative** (ne-GAY-tiv) **operator.** Thus, the word 'not' is a negative operator. Other negative operators in English are 'it is not the case that', 'it is false that', and 'no', to name a few.

In logic there is only one negative operator: the symbol '~', called **tilde** (TIL-deh). Whenever the tilde is placed immediately to the left of a statement, abbreviated or otherwise, the result is a compound statement, a negation, whose component is the statement that the tilde is immediately to the left of. The tilde, like the dot, is a pure truth-functional operator which has no exact synonym in English. Rather, it corresponds to all of the various negative operators in English, and when performing logical translation it is standard practice to translate all negative operators by the tilde.

For example, the logical translation of (2c) from §2.2

2c. $U \cdot$ it is not the case that I

can be completed by translating the negative operator 'it is not the case that' by the tilde, to produce

2d. $U \cdot \sim I$

Similarly, the logical translation of (1) might proceed as follows:

1. Harvey is not married.

paraphrases to

1a. It is not the case that Harvey is married.

which abbreviates to

1b. It is not the case that M.

which translates to

1c. $\sim M$

The truth-table definition of the tilde is:

p	$\sim p$
T	F
F	T

When reading the symbolic language aloud, it is conventional to read the '~' as "tilde," "not," or "it is false that." "Not" is perhaps most common.

There is an obvious but important difference between the conjunctive operator and the negative operator. The negative operator is a **unary** operator —it operates upon a *single* element (proposition) to negate it; the conjunctive operator is a **binary** operator—it operates upon a *pair* of elements (propositions) to conjoin them. In formal logic, a unary operator is normally placed immediately to the left of the element it operates on, with punctua-

tion (discussed in the next section) to show how much of the following formula it governs. However, a binary operator is normally placed between the elements it operates on, with punctuation to show its "scope" in both directions.

§2.5 Punctuation

Punctuation marks belong almost uniquely to written languages. They serve in lieu of the inflections of voice and tone which operate in spoken language to inform the listener of how the speaker's phrases are jointed, of what is to be grouped with which. There are many notorious examples of the ambiguity that can result when we are not sure of how the parts of a sentence are to be grouped together. For example, the sentence

1. I saw him jump through the keyhole.

can express two completely different propositions (one absurd, the other plausible) if we group its elements in different ways:

1a. I saw him (jump through the keyhole).

1b. I saw him jump (through the keyhole).

Similarly, the sentence

2. Irishmen who are cowards are fortunately rare in prizefighting.

says one thing without internal punctuation and something quite different when punctuated as

2a. Irishmen, who are cowards, are fortunately rare in prizefighting.

In logic, as elsewhere, it is important to be able to avoid such ambiguities. Consider, for example, the sentence

3. It is false that there is life on Mars and there is life on Venus.

In the absence of punctuation, there is no way to tell whether the negative operator is applied to the whole conjunction or only to its lefthand conjunct. The same is true for the logical translation of (3):

3a. $\sim M \cdot V$

The ambiguities of (3) and (3a) are easily overcome, however, with an adequate system of punctuation and an appropriate set of conventions governing its application.

The only punctuation marks used in formal logic are parentheses of various shapes, such as curved parentheses ( ), square brackets [ ], and braces { }.* And while the conventions governing their use are much more easily

*Some alternative methods of punctuation are presented in Appendix D.

seen than said, a short explanation may nevertheless be helpful. In giving this, it will be necessary to revise a bit of terminology slightly.

The notion of a "logical unit" is as follows: a simple statement is a logical unit; a negation is a logical unit; a compound statement enclosed by a pair of parentheses is a logical unit. Truth-functional operators attach to logical units (single ones in the case of negation; pairs of them in the case of other operators) to form compounds. When a negative operator is attached to a logical unit to form a negation, what is negated is the *unit*, rather than anything concealed inside the unit (if such there be). Similarly, when a conjunctive operator is attached to a pair of logical units to form a compound, what are conjoined are the *units*, not anything sealed up inside the units.

Parentheses serve to package up compounds into units. Just as the wrappings are part of the package, so the parentheses are part of the unit. The general rules of logical punctuation may be stated as follows: a tilde (negative operator) applies to the whole of the first logical unit to its right, and nothing more (or less); any other operator applies to the whole of the two logical units on its immediate right and left, and nothing more (or less).

Given these conventions, (3a) as it stands is a conjunction, one of whose conjuncts is the negation 'It is false that there is life on Mars'. Thus, (3a) will be true only if there is life on Venus but none on Mars. The negation of the whole (compound) statement 'There is life on Mars and there is life on Venus' is written, not as (3a), but as

　　3b. $\sim(M \cdot V)$

which will be true if either or both planets are devoid of life—that is, if $(M \cdot V)$ is false.

§2.6 Mating Parentheses

As statements in the symbolic language become more complex, the punctuation tends to become more difficult to follow; it is sometimes not easy to tell, just by looking, which lefthand parenthesis mates with which righthand one. Should this difficulty arise, it can easily be overcome by "mating" the pairs—joining them with a line drawn from the one to the other. Properly done, this will provide a clear picture of the way the various components are nested one inside the other.

To mate parentheses, proceed along the formula* from left to right until you come to a righthand ("closing") parenthesis; backtrack to the last lefthand ("opening") parenthesis you passed, and mate the two. Then continue on through the formula. Each righthand parenthesis mates with the closest unmated lefthand parenthesis to its left. For example, consider the formula

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*A *formula* is any series of (one or more) parentheses, letters, and/or operators.

$$\sim((A \cdot \sim(B \cdot C)) \cdot (((B \cdot D) \cdot \sim(A \cdot \sim D)) \cdot \sim B))$$

Proceeding from left to right, the first closing parenthesis we come to is the one after 'C'; we backtrack to the last opening parenthesis we passed (the one before the first 'B') and mate the two:

$$\sim((A \cdot \sim(B \cdot C)) \cdot (((B \cdot D) \cdot \sim(A \cdot \sim D)) \cdot \sim B))$$

Continuing rightwards, the next closing parenthesis (the second one after 'C') is mated to the last unmated opening parenthesis (the one before 'A') thus:

$$\sim((A \cdot \sim(B \cdot C)) \cdot (((B \cdot D) \cdot \sim(A \cdot \sim D)) \cdot \sim B))$$

and so on throughout the formula, mating each closing parenthesis with the last unmated opening one until all are mated:

$$\sim((A \cdot \sim(B \cdot C)) \cdot (((B \cdot D) \cdot \sim(A \cdot \sim D)) \cdot \sim B))$$

In this procedure there are two things to watch out for. (a) If any mating lines cross each other, you have done something wrong. (b) If any parentheses turn out to be "unmatable"—if you run out of closing (or opening) parentheses before using up all the opening (or closing) ones—then the formula is incorrectly written and must be changed, either by the addition of more parentheses or the deletion of excess ones.

Using alternating parentheses of different shapes can be helpful in laying out the structure of a complex formula; for example, the structure of the above is clearer when written

$$\sim\{[A \cdot \sim(B \cdot C)] \cdot [(\{B \cdot D\} \cdot \sim\{A \cdot \sim D\}) \cdot \sim B]\}$$

However, it is not a logical requirement that different-shaped parentheses be used. In the present book formulae are written sometimes with different-shaped parentheses and sometimes with only round ones.

When reading the symbolic language aloud, it is conventional to read lefthand parentheses as "pren" ("bracket," "brace," and so on) and righthand parentheses as "close pren" ("close bracket," "close brace," and so on). For example, the statement

　　1. $\sim(M \cdot V) \cdot J$

would be read aloud as

　　1a. Not pren em and vee close pren and jay.

Similarly, the statement

　　2. $\sim((M \cdot V) \cdot J)$

would be read aloud as

　　2a. Not pren pren em and vee close pren and jay close pren.

Exercises I

Give a logical translation of each of the following statements. Paraphrase when necessary, abbreviate all simple components, translate negative operators by the tilde, conjunctive operators by the dot, and supply punctuation as needed. You need not attempt translation further than this.

 1. Percy kicked his sister, but she didn't scream.
☐ 2. Percy didn't kick his sister, in spite of the fact that she was screaming.
 3. It's false that Percy kicked his sister even though she was screaming; but he did kick her even though she *wasn't* screaming.
☐ 4. If Percy didn't kick his sister and she didn't scream, then either my ears aren't very good or somebody is lying.
 5. It's not the case that it's false that Percy's sister didn't scream.
☐ 6. While Percy's sister didn't scream, he kicked her nevertheless.
 7. Percy didn't kick his sister, since she didn't scream.

Exercises II

Supposing that A and B are both true statements, and that X and Y are both false statements, determine which of the following compound statements are true. It may be helpful to mate the parentheses in some of the more complex examples.

☐ 1. $A \cdot X$
 2. $\sim A \cdot X$
 3. $\sim(A \cdot X)$
☐ 4. $A \cdot \sim Y$
 5. $\sim(A \cdot \sim Y)$
 6. $A \cdot B$
☐ 7. $\sim \sim A \cdot B$
 8. $\sim(\sim A \cdot B)$
 9. $\sim(\sim A \cdot \sim B)$
☐ 10. $\sim \sim(A \cdot \sim B)$
 11. $\sim[(A \cdot X) \cdot \sim(B \cdot Y)]$
 12. $\sim \sim[\sim(A \cdot B) \cdot \sim(\sim X \cdot \sim Y)]$
☐ 13. $A \cdot \sim A$
 14. $\sim(Y \cdot \sim Y)$
 15. $\sim \sim \sim \sim \sim(\sim Y \cdot B)$
☐ 16. $\sim(A \cdot \sim\{B \cdot \sim[X \cdot \sim(Y \cdot \sim A)]\})$
 17. $A \cdot [B \cdot (X \cdot Y)]$
 18. $A \cdot [(B \cdot X) \cdot Y]$
☐ 19. $(A \cdot B) \cdot (X \cdot Y)$
 20. $[(A \cdot B) \cdot X] \cdot Y$
 21. $[A \cdot (B \cdot X)] \cdot Y$

chapter 3

DISJUNCTION; BRACKETING AUXILIARIES; DOMINANCE AMONG OPERATORS

§3.1   Disjunction

As we have seen, a **conjunction** is a truth-functional compound which is true when both of its components are true and false otherwise. Another sort of truth-functional compound is the **disjunction** (also called **alternation**). A disjunction is a compound which is true if one or the other of its components is true, and false otherwise. The components of a disjunction are called its **disjuncts**. An operator which produces a disjunction when appropriately applied to a pair of statements is called a **disjunctive operator**. Perhaps the most commonly used English disjunctive operator is the word 'or', as in the statement

1. This soup contains too much salt or too little water.

There are two different types of disjunction, the "strong" or "exclusive" disjunction and the "weak" or "nonexclusive" disjunction. A **strong disjunction** is true if *one* of its disjuncts is true, but false otherwise (that is, is false if both disjuncts are false, and is false if they are both true). A **weak disjunction** is true if *one or both* of its disjuncts are true, but false otherwise (is false only if both disjuncts are false). English contains operators of both types. The most obvious example of a strong disjunctive operator is the expression 'or . . . but not both', as in the statement

2. We will go to the beach or to the mountains, but not both.

Similarly, the most obvious example of a weak disjunctive operator is the expression 'or . . . or both', as in the statement

3. Jones is sick or incompetent, or both.

But some operators are ambiguous or indifferent as to the strong/weak distinction. The word 'or' is a case in point. Is (1) above a strong disjunction or a weak one? Intuitively, we should probably regard it as a weak disjunction; that is, if the soup were both oversalted and underwatered we would

not regard that as grounds for calling (1) false. Nevertheless, there are conceivable circumstances in which 'or' might be used to form a strong disjunction. But it is standard practice in logic to treat all disjunctions as *weak* disjunctions unless they are clearly and explicitly strong ones—that is, unless they contain the operator 'or . . . but not both' or something very similar. In this connection two points must be made.

First, the character of the disjuncts does *not* dictate the sense in which a disjunctive operator is being used. There is a tendency to suppose that in a statement such as

4. Jones is, at this moment, in Spain or he is in Tibet.

the word 'or' must have the sense of strong disjunction, since it is impossible for both disjuncts to be true (Jones cannot simultaneously be in Spain and in Tibet). But this supposition is unwarranted, for, in the first place, the *fact* that both disjuncts cannot be true in no way shows that the *operator* explicitly denies their conjoint truth; and in the second place, the fact that both disjuncts *cannot* be true would seem to indicate that (4) is a perfect place to employ *weak* disjunction, since the situation which would otherwise be ruled out by strong disjunction (mutual truth of both disjuncts) is ruled out *de facto* by the nature of the case, without the need for any disclaimers to that effect.*

Second, the expression 'either . . . or' is *not* equivalent to 'or . . . but not both'. It is sometimes supposed that the auxiliary 'either' produces a strong disjunctive operator. However, the function of 'either' in 'either . . . or' is not to generate strong disjunction but to serve as a kind of bracket, indicating the beginning of the first disjunct of the disjunction. It is a verbal punctuation mark. For example, the statement

5. Old Williams is preparing to retire and Smith will become chairman or the company is headed for ruin.

as it stands is ambiguous; but suitable applications of 'either' eliminate the ambiguity in the same way that parentheses would. One interpretation of (5) is

5a. Either Old Williams is preparing to retire and Smith will become chairman or the company is headed for ruin.

The other interpretation of (5) is

5b. Old Williams is preparing to retire and either Smith will become chairman or the company is headed for ruin.

and both (5a) and (5b) are unambiguous. (5a) is clearly a *disjunction* whose first disjunct is a compound statement about Old Williams and Smith; (5b) is clearly a *conjunction* whose second conjunct is a compound statement

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*This point is aptly made by Quine, *Methods of Logic*, 3d ed., p. 11.

about Smith and the company. So far as the strong/weak distinction goes, 'either . . . or' has the same status as 'or'.

In logic there is only one disjunctive operator, the symbol '$\lor$', called **wedge**. The wedge is a weak disjunctive operator. Whenever it is placed between two statements, abbreviated or otherwise, the result is a weak disjunction whose disjuncts are the statements that the wedge occurs between. Like the dot and the tilde, the wedge is a pure truth-functional operator with no exact synonym in English. It is standard practice to translate all weak disjunctive operators, and all *indifferent* disjunctive operators, by means of the wedge. (The procedures for translating strong disjunctive operators are given later on.) For example, the logical translation of (1) above will proceed as follows:

1. This soup contains too much salt or too little water.

becomes, by paraphrase,

1a. This soup contains too much salt or this soup contains too little water.

which abbreviates to

1b. *M* or *L*

which, translating 'or' by the wedge, becomes

1c. $M \lor L$

When reading the symbolic language aloud, the wedge is conventionally pronounced as "wedge" or as "or." The truth-table definition of the wedge is

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

which says that a disjunction formed with the wedge is false when both disjuncts are false, and true otherwise.

§3.2   Bracketing Auxiliaries

As has already been noted in the case of 'either . . . or . . .', some English truth-functional operators contain what might be called a **bracketing auxiliary**: a word which delimits the "left scope" of the operator. For example, in a sentence containing 'either . . . or . . .', the words 'either' and 'or' bracket the first (lefthand) disjunct of the disjunction: the first disjunct consists of everything between the auxiliary 'either' and the primary operator-word 'or'; and the second disjunct will normally be everything after the 'or',

unless punctuation indicates otherwise. In such a case it is tacitly understood that operators coming *before* the bracketing auxiliary govern the entire compound (for example, the entire disjunction) rather than merely its first element.

This means that whenever there is a bracketing auxiliary, (i) it may initially be translated as an opening bracket whose mate falls just before the primary operator-word, and (ii) punctuation may be inserted to assure that any operators *before* the auxiliary govern the entire compound. Following this procedure, the logical difference between the two statements

1. Either it's not the case that Smith will become chairman or the company will be ruined.
2. It's not the case that either Smith will become chairman or the company will be ruined.

is quickly brought out as the difference between

1a. ((It's not the case that Smith will become chairman) or the company will be ruined).

and

2a. It's not the case that ((Smith will become chairman) or the company will be ruined).

which translate fully as

1b. $(\sim C \vee R)$
2b. $\sim (C \vee R)$

Another example of a bracketing auxiliary is the phrase 'not only' in the conjunctive operators 'not only . . . but . . .' and 'not only . . . but also . . .'.*
As in the previous example, the difference between the two statements

3. It's false that not only will Smith become chairman but the company will be ruined.
4. Not only is it false that Smith will become chairman but the company will be ruined.*

comes out clearly, in translation, as the difference between

3a. It's false that ((Smith will become chairman) but the company will be ruined).
4a. (It's false that (Smith will become chairman)) but the company will be ruined.

which translate fully as

3b. $\sim (C \cdot R)$
4b. $\sim C \cdot R$

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*An oddity of English grammar is that 'not only' usually leads to an inversion in the normal word order ('Smith will decide' but 'Not only will Smith decide . . .'). Such inversions are irrelevant to logic and may easily be eliminated by paraphrase in the course of translation.

As a further illustration of the translation of bracketing auxiliaries, the two statements

5. Not only will Old Williams retire unless Smith advises against it, but the stockholders are prepared to rebel.
6. Old Williams will retire unless not only Smith advises against it but the stockholders are prepared to rebel.

differ in precisely the same way as the following:

5a. (Old Williams will retire unless Smith advises against it) but the stockholders are prepared to rebel.
6a. Old Williams will retire unless (Smith advises against it but the stockholders are prepared to rebel).

or, fully translated,

5b. $(W \vee A) \cdot S$
6b. $W \vee (A \cdot S)$

When paraphrasing English statements as a preparation for translating them, it is sometimes quite helpful to supply bracketing auxiliaries if they do not appear in the original. Also, it sometimes happens that when the auxiliary is given, the primary operator-word is suppressed, as in the statement

7. Not only is Smith incompetent, he is insane.

In such cases it is also helpful (if not downright necessary) to supply the missing operator-word, prior to translating. In the lists of English operators given as Translation Aids, the operators involving bracketing auxiliaries are given with a three-dot ellipsis ('either . . . or'; 'both . . . and').

Bracketing auxiliaries, and the operators they go with, may be mated in the same way as parentheses. Doing this before attempting formal punctuation will usually show quite graphically how the various components of the statement are nested together. First, abbreviate all the simple components. Then go through the sentence from left to right and mate each operator with the first grammatically appropriate and unmated bracketing auxiliary to its left.* The result will be a picture of how the sentence is articulated. For example, the sentence

8. Either both A and B or both either C or not only D but E and F.

contains a number of bracketing auxiliaries. When these are mated to the operators they go with, the result is

8a. Either both A and B or both either C or not only D but E and F.

which shows that 'both A and B' is the first half of a disjunction, since it is sandwiched between a corresponding 'either' and 'or', so that the correct

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*Except 'if' and 'then'; they can't be mated thus mechanically.

punctuation is

    8b. Either (both A and B) or both either C or not only D but E and F.

It also shows that 'either C or not only D but E' is the first half of a conjunction, since it is between a corresponding 'both' and 'and', so that the punctuation is

    8c. Either (both A and B) or both (either C or not only D but E) and F.

with what follows the 'and' being the second conjunct, so that the whole conjunction may be packaged up as

    8d. Either (both A and B) or (both (either C or not only D but E) and F).

Similarly, 'D' is the first element of a conjunction (since it is between 'not only' and 'but'), and what follows 'but' is the second element, so that the whole conjunction brackets as

    8e. Either (both A and B) or (both (either C or (not only D but E)) and F).

which may now be translated easily into

    8f. $(A \cdot B) \vee ((C \vee (D \cdot E)) \cdot F)$

    A final remark about paraphrasing and punctuating: the whole process of giving paraphrases hinges upon *understanding* the English statements being paraphrased; there can be no formal rules for paraphrasing, since a given sentence may express different statements on different occasions, and when that happens different paraphrases may be required. Frequently, a statement which would be totally ambiguous in isolation has a meaning which is quite clear given the context of its utterance. However, when the context is not provided, or when there is otherwise no way of telling which of its possible meanings a given sentence may have, there are three alternatives open: (1) either ignore the statement altogether and go do something else, or (2) arbitrarily select one of the various possible meanings the sentence might have and translate in accordance with that, or (3) give different translations corresponding to *each* of the various possible meanings, and subject each of these in turn to whatever logical purposes you have in mind. The first of these alternatives is not available to the student of logic who is doing exercises for the purpose of learning; the third is normally too time-consuming for the student who is simply doing exercises. Hence, *unless otherwise indicated,* the second alternative is the one to take. It must be emphasized, however, that this is a last resort. One should never be too quick to judge a statement hopelessly ambiguous and give it an arbitrary interpretation. As an illustra-

tion, the two statements

    9. Old Williams will retire tomorrow; and Smith will become chairman or the company is headed for ruin.

    10. Old Williams will retire tomorrow and Smith will become chairman, or the company is headed for ruin.

contain no bracketing auxiliaries, but this doesn't mean that we are unable to distinguish between them, or to tell how they differ. Here, the English punctuation marks serve to indicate how the various clauses are grouped, telling us that the two are to be punctuated, respectively, as

    9a. $W \cdot (C \vee R)$
    10a. $(W \cdot C) \vee R$

without the need for any arbitrary decisions.

TRANSLATION AID

The following are some disjunctive operators in English.

or	otherwise
either ... or ...	with the alternative that
or else	unless
or, alternatively	

Exercises I

Give a logical translation of each of the following. Paraphrase as necessary, abbreviate all simple components, translate disjunctive operators by the wedge, conjunctive operators by the dot, negative operators by the tilde, and supply punctuation as needed. You need not attempt translation further than this.

☐ 1. Percy has taken the car without telling us, or else someone has stolen it and we must inform the insurance company.

  2. I shall blot out the sun and destroy your crops unless you release me immediately and return the magic ring to me.

☐ 3. You will have to take the shortcut home; otherwise you'll be late and either you won't get any supper or you'll have to eat leftovers.

  4. You will have to take the shortcut home; otherwise either you'll be late and you won't get any supper or you'll have to eat leftovers.

☐ 5. If it doesn't rain or snow tomorrow and they get the car fixed in time, then we'll go to the mountains or to the beach and do some surfing.

  6. If it rains and snows tomorrow or they don't get the car fixed in time, then we won't go to the beach or have a picnic.

Exercises II

Supposing that A and B are both true statements, and that X and Y are both false statements, determine which of the following compound statements are true:

☐ 1. $\sim(A \vee X)$

  2. $\sim A \vee \sim X$

3. $A \lor (X \cdot Y)$

□ 4. $(A \lor X) \cdot Y$

5. $(A \cdot B) \lor (X \cdot Y)$

6. $(A \lor B) \cdot (X \lor Y)$

□ 7. $A \cdot [X \lor (B \cdot Y)]$

8. $X \lor [A \cdot (Y \lor B)]$

9. $X \lor \sim X$

□ 10. $A \lor \sim A$

11. $\sim\{\sim[\sim(A \cdot \sim X) \cdot \sim A] \cdot \sim X\}$

12. $[(A \cdot X) \lor \sim B] \cdot \sim[(A \cdot X) \lor \sim B]$

□ 13. $[(X \lor A) \cdot \sim Y] \lor \sim[(X \lor A) \cdot \sim Y]$

14. $\{X \lor [Y \lor \sim(A \lor X)]\} \lor B$

§3.3 Translating "Strong" Disjunctions

When it becomes necessary to translate a strong disjunction, we may do so with the symbolism already at our disposal, with no need for anything new. Strong disjunctions assert "one or the other, but not both," and this is exactly the way they are translated. The strong disjunction

    1. Harvey visited Washington or Baltimore, but not both.

becomes, by repeated paraphrase:

    1a. Harvey visited Washington or Harvey visited Baltimore, but Harvey did not visit both Washington and Baltimore.

    1b. Harvey visited Washington or Harvey visited Baltimore, but it is not the case that Harvey visited both Washington and Baltimore.

    1c. Harvey visited Washington or Harvey visited Baltimore, but it is not the case that both Harvey visited Washington and Harvey visited Baltimore.

which abbreviates to

    1d. W or B, but it is not the case that both W and B.

which translates, step by step, as

    1e. $(W \lor B)$ but it is not the case that both W and B

    1f. $(W \lor B)$ but it is not the case that $(W \cdot B)$

    1g. $(W \lor B) \cdot$ it is not the case that $(W \cdot B)$

    1h. $(W \lor B) \cdot \sim(W \cdot B)$

(1h) is a conjunction whose first conjunct asserts "one or the other" and whose second conjunct asserts "not both"; thus (1h) asserts "one or the other, but not both." *Any* strong disjunction may be translated in this same way—that is, as a conjunction asserting "one or the other" and "not both."

Exercises III

Give a logical translation of the following statements.

    1. Either Old Williams will retire or Smith will become chairman, but not both.

□ 2. We'll go to the mountains and have a picnic or to the beach and do some surfing, but not both.

    3. Either we'll go to the mountains and have a picnic, or else we'll go to the beach and do some fishing or some surfing but not both.

□ 4. Either Old Williams won't retire or Smith won't become chairman, but not both.

    5. Either we'll go to the mountains or have a picnic or both, or we'll go to the mountains or have a picnic but not both.

§3.4   Dominance and Subordination among Operators

When an operator attaches to a statement or a pair of statements to form a compound, the statement or statements to which the operator attaches are the components of that compound. However, for the sake of brevity, it is sometimes convenient to refer to them as components *to the operator* which generates the compound. Speaking in this loose fashion, we might say that in the statement

    1. $\sim(A \lor B) \cdot C$

the components to the dot are the statements '$\sim(A \lor B)$' and 'C', while the components to the wedge are the statements 'A' and 'B', and the component to the tilde is the statement '$(A \lor B)$'.

One operator **dominates** another if the other occurs in a component to the one. Thus, in (1), the tilde dominates the wedge, and the dot dominates the tilde (and also the wedge). If one operator dominates another, the second is said to be **subordinate** to the first. In (1), the tilde is subordinate to the dot, and the wedge is subordinate to the tilde (and also to the dot).

An operator is the **dominant operator** in a formula if it dominates all other operators in the formula. In (1), the dominant operator is the dot. But in the first conjunct of (1) the dominant operator is the tilde.

Clearly, there are different levels of domination—a "pecking order" among the operators in compound statements. One operator **directly dominates** another if the second is the dominant operator in a component to the first. If one operator directly dominates another, the second is **directly subordinate** to the first. Thus, in (1), the dot directly dominates the tilde. The dot does *not* directly dominate the wedge, since the wedge is not the dominant operator in either of the components of (1). (Remember, '$(A \lor B)$' is *not* a component

of (1)—is not a component to the dot—since '$(A \lor B)$' is not one of the conjuncts of the conjunction generated by the dot.)

If one operator dominates another, but does not directly dominate it, then it is a case of *indirect* domination. In (1), the dot indirectly dominates the wedge. We could, if necessary, coin a terminology to cover different levels of indirect domination. For example, we might say that in

2. $\sim(X \lor \sim Y) \cdot Z$

the dot exercises *first-level* indirect domination over the wedge and *second-level* indirect domination over the tilde in '$\sim Y$' (since the dot indirectly dominates the wedge, and the wedge dominates the tilde in '$\sim Y$'). But this terminology is ordinarily unneeded at the level of elementary logic.

A formula is not properly written in the symbolic language unless it contains a **dominant operator**, whose dominance is clearly shown by the punctuation.* For example, the formula

3. $A \cdot B \lor C$

is not a properly written statement in the symbolic language, since there is no way to tell whether its dominant operator is the dot or the wedge. And, perhaps needless to say, a formula is not properly written in the symbolic language unless its component or components are properly written.

The dominant operator in a statement defines the logical character of the statement. For example, if the statement contains no operators, then it is **simple**; if its dominant operator is a tilde, then it is a **negation**; if its dominant operator is a dot, then it is a **conjunction**; and so on. Hence, we might describe (1) by saying: it is a conjunction, whose second conjunct is simple and whose first conjunct is a negation whose negate is a disjunction whose disjuncts are simple. Another way of stating the points in the preceding paragraph is that a statement is not properly written in the symbolic language unless it can be given a complete description of this sort. Thus (3) is not a properly written statement, since we are stymied, at the very beginning, as to whether it is a conjunction or a disjunction.

Exercises IV

Pick out and circle the dominant operator in each of 1–14, Exercises II.

Exercises V

Give a complete logical description of each of statements 1–7 in Exercises II. As an illustration:

(1) A negation whose negate is a disjunction whose disjuncts are simple.

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*The only exceptions are simple statements which contain no operators at all.

Exercises VI

Do the same for 8–14. As an illustration:

(12) A conjunction whose first conjunct is
    a disjunction whose first disjunct is
        a conjunction whose conjuncts are simple, and
    whose second disjunct is
        a negation whose negate is simple; and
  whose second conjunct is
    a negation whose negate is
        a disjunction whose first disjunct is
            a conjunction whose conjuncts are simple, and
        whose second disjunct is
            a negation whose negate is simple.

chapter 4

CONDITIONALS

§4.1 Conditionals

In everyday discourse we often find it worthwhile to assert that *if* some statement is true, then so is another. When such an assertion is made, the result is a compound statement, formed with the help of an operator such as 'if...then...'. Compound statements of this kind are called **conditional** statements, and the operator which generates the compound is called a **conditional operator**.

The logical treatment of conditionals is a more complicated affair than the treatment of conjunctions and disjunctions. For one thing, many conditionals are not even truth-dependent compounds, a matter discussed at some length later on. But even those conditionals which are truth-dependent compounds present difficulties when we try to fit them into the rigid structure of formal logic. Let us begin by considering some examples.

1. If Harvey traveled to Spain, then he crossed the Atlantic Ocean.

First, it should be noted that conditionals are not "symmetrical" as conjunctions and disjunctions are: the order in which the components of a conditional occur makes a difference to what is said. Statement (1) says something quite different from

2. If Harvey crossed the Atlantic Ocean, then he traveled to Spain.

even though they have the same components. Thus, instead of using a single word to designate the components of a conditional (as with the *conjuncts* of a conjunction or the *disjuncts* of a disjunction), a different designation is used for each of the two components. One of them is called the **antecedent** (or the **protasis**) of the conditional, and the other is called the **consequent** (or the **apodosis**) of the conditional. In a conditional formed with the operator 'if...then...', for example, the component occurring between the 'if' and the 'then' is the antecedent; the component occurring after the 'then' is the consequent. In (1) above, the antecedent is the statement 'Harvey traveled to Spain' and the consequent is 'He crossed the Atlantic Ocean'.

The first prerequisite for fitting conditionals into truth-functional logic is to decide what their *truth-conditions* are. Until this is done, it will not even be possible to translate them, since the purpose of logical translation is to swap English statements for formulae in the symbolic language which have the same truth-conditions.

It will be easiest to begin by seeking out the conditions under which a conditional statement is *false*. Normally, the way in which we *show* a conditional to be false is by showing that, although it has a true antecedent, it nevertheless has a false consequent. For example, if we could demonstrate that although Harvey did travel to Spain he did not cross the Atlantic (perhaps he went, and returned, via China), this would be sufficient to show that (1) is false. Similarly, the conditional

3. If that candle is left out in the sun, it will melt.

may be proven false by leaving the candle out in the sun and observing its failure to melt. This would prove (3) false by showing that even though its antecedent is true, its consequent is nevertheless false. In general, *whenever* the antecedent of a conditional is true but its consequent is false, then the conditional is false. This may be stated as

CONDITION ONE. A conditional is **false** if it has a true antecedent but a false consequent.

Do conditional statements have any other falsity-conditions besides this one? There is no *definitely* established answer; the semantics of conditional statements is still a subject of considerable study. But it is quite certain that there is no other *truth-functional* falsity condition for statements of this type. Hence, for purposes of formal logic Condition One is taken as the *only* falsity condition, and conditional statements are regarded as true unless they have a true antecedent and false consequent.

This decision is not *purely* arbitrary. It accords to a large extent with our standard attitude toward conditional statements. For example, from Condition One it follows that if a conditional is true, and has a true antecedent, it must also have a true consequent. If any conclusion is to be drawn from this, it is that one way for a conditional to be *true* is for it to have a true antecedent and also a true consequent. And this adheres very closely to one of our standard ways of verifying conditionals. Thus, (3) might be proven true by leaving the candle in the sun and seeing it melt. In that case, the person uttering (3) might say, "See? I was right." And we would no doubt admit that he was indeed right; what he said was true. We observe that the antecedent of his claim is true, and that its consequent is also true, and thus we admit the correctness of the claim. This can be phrased formally as

CONDITION TWO. A conditional is **true** if it has a true antecedent and also a true consequent.

In a similar fashion, the way we operate with conditionals in normal

discourse also provides a basis for saying that a conditional can be true when its *consequent* is false, provided that its antecedent is also false. This may be brought out in several different, but related, ways.

First, when we accept a conditional as true we do not thereby commit ourselves to the truth of its consequent (for example, if we accept (1) as true we do not thereby commit ourselves to the belief that Harvey has ever crossed the Atlantic). Our commitment is only that its consequent is true *if* its antecedent is. A later discovery that its consequent is false will not, thereby, refute the original conditional. The discovery that a conditional's consequent is false only shows that its antecedent must also be false. Given the conditional statement

4. If Clyde is at home then he left the porch light on.

the discovery that Clyde's porch light is off will not entitle us to infer that (4) is false, but only that Clyde is not at home. It is an interesting (and logically important) feature of conditionals that they cut both ways. From Condition One it follows that a true conditional with a true antecedent must also have a true consequent; and likewise a *true* conditional with a false consequent must also have a false antecedent. (Otherwise the conditional would not be *true*.) And just as we sometimes confirm a conditional [as in the case of (3)] by first ascertaining that its antecedent is true and then discovering the truth of its consequent, so also we sometimes confirm a conditional the other way around, by first ascertaining the falsity of its consequent and then discovering the falsity of its antecedent. For instance, we might confirm (4) by first phoning Clyde (to make sure he's home) and then driving past his house and observing that the porch light is on. But we might also confirm (4) by first noticing that his porch light is off, and then breaking into his house to discover that he is absent. In either case, the person asserting (4) might say, "See? I was right." And in either case, we would no doubt agree.

In accepting a conditional as true, we are agreeing that *if* its antecedent is true, *then* its consequent is also true. But this is the same as agreeing that *if* its consequent is false, *then* so is its antecedent. Agreeing to (4) is the same as agreeing to

5. If Clyde didn't leave the porch light on then he's not at home.

In both cases, we are denying that Clyde is home with the porch light off. Two statements related as (4) and (5) are said to be *contrapositives*. The contrapositive of a conditional is formed by exchanging the two components and then negating them. Thus the antecedent of (5) is the negation of the consequent of (4), and the consequent of (5) is the negation of the antecedent of (4). A conditional and its contrapositive are, to all intents and purposes, equivalent statements. In normal discourse we quite often use them interchangeably, as mere stylistic variants of one another. For example, instead

of (1) we might just as easily have said

6. If Harvey didn't cross the Atlantic, then he didn't travel to Spain.

And insofar as two statements are equivalent, they have exactly the same truth-conditions. Thus (4) and (5) have the same truth-conditions [as do (1) and (6)]. But each of the components of (5) is the negation of a component of (4), so when both components of (4) are true, both components of (5) are false. But also when both components of (4) are true, (4) is true—per Condition Two. And when (4) is true, (5) is true, since they are equivalent. Therefore, when both components of (5) are false, then (5) is true.

In general, the mutual falsity of the components of a conditional is on a logical par with their mutual truth. We can state this formally as

CONDITION THREE. A conditional is **true** if it has a false consequent and also a false antecedent.

This leaves us with one troublesome case: that of the conditional whose consequent is true but whose antecedent is false. Here, there are no standards within normal discourse for us to appeal to. If a conditional turns out to have a true antecedent but a false consequent, we accept that as proof that the conditional is false. If a conditional turns out to have a true consequent along with a true antecedent, or a false antecedent along with a false consequent, we (may) accept that as proof that the conditional is true. However, when a conditional turns out to have a false antecedent but a true consequent, we do not regard it either as true or as false, but simply as *pointless*. There is nothing in common practice to tell us which truth-value to ascribe to such conditionals, since it is just here that common practice is silent on the matter. Appeals to contrapositives are equally useless, since if a conditional has a false antecedent and a true consequent, so does its contrapositive. Thus, if a truth-value is to be ascribed to such conditionals, we are forced to choose one arbitrarily. In doing this, it is of some comfort to know that whatever choice we make, it cannot *conflict* with common usage, for the very good reason that common usage has nothing to say.

Thus, for purposes of logic, it is stipulated that a conditional with a false antecedent and a true consequent is to be regarded as *true*, which allows us to assert

CONDITION FOUR. A conditional is **true** if it has a false antecedent and a true consequent.

Although this choice has some curious-sounding consequences, which will be touched upon later, it is justified by the fact that it is the only choice which will allow conditionals into truth-functional logic.*

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*If the opposite choice were made—if we elected to regard the "pointless" cases as *false*— this would have the effect of eliminating the distinction between conditionals and what are called *biconditionals* (cf. Chapter 5).

§4.2 Material Conditionals

A conditional statement which is true except when its antecedent is true and its consequent is false is called a **material conditional**. Thus, the effect of stipulating Condition Four is that every (truth-dependent) conditional will be interpreted, for logical purposes, as a material conditional.

Two facts about material conditionals have led some logicians to regard them as in some way "paradoxical." These facts are: (1) a material conditional whose antecedent is false will be true irrespective of the truth-value of its consequent, and (2) a material conditional whose consequent is true will be true irrespective of the truth-value of its antecedent. These facts are sometimes referred to as "the paradoxes of material implication."*

In logic, there is only one conditional operator; the symbol '⊃', called **horseshoe**. Whenever it is placed between two statements, abbreviated or otherwise, the result is a material conditional, whose antecedent is the statement to the left of the horseshoe, and whose consequent is the statement to the right of the horseshoe. The horseshoe, like the dot, wedge, and tilde, is a pure truth-functional operator which has no exact synonym in English. Rather, it is used to translate all of the various English conditional operators. The truth-table definition of the horseshoe is:

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

As illustrations of the way the horseshoe is used, the statement

1. If Harvey traveled to Spain, then he crossed the Atlantic Ocean.

becomes, by abbreviation,

1a. If S then C

which, translating 'if . . . then . . .' by the horseshoe, becomes

1b. $S \supset C$

Similarly, the statement

2. If Harvey visited Spain and Portugal, then he crossed the Atlantic or went the long way around.

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*Material conditionals are sometimes called "statements of material implication," and practitioners of logic sometimes say that the antecedent of a material conditional "materially implies" its consequent.

paraphrases to

2a. If Harvey visited Spain and Harvey visited Portugal, then Harvey crossed the Atlantic or Harvey went the long way around.

which abbreviates to

2b. If S and P then C or L.

which translates, step by step, as

2c. If $(S \cdot P)$ then C or L
2d. If $(S \cdot P)$ then $(C \vee L)$
2e. $(S \cdot P) \supset (C \vee L)$

As a slightly more complex example, the statement

3. If Harvey visited Spain, then if he didn't cross the Atlantic then he went the long way around.

paraphrases to

3a. If Harvey visited Spain, then if it's not the case that Harvey crossed the Atlantic then Harvey went the long way around.

which abbreviates to

3b. If S then if it's not the case that C then L.

which translates, step by step, as

3c. If S then if $\sim C$ then L
3d. If S then $(\sim C \supset L)$
3e. $S \supset (\sim C \supset L)$

When reading the symbolic language aloud, we customarily read the '⊃' as "horseshoe" or as "if . . . then . . .," or sometimes as "implies," though the last can be misleading if taken to mean what the English word 'implies' usually means.

TRANSLATION AID

Unfortunately for the logical translator, English conditionals do not always come with the antecedent conveniently placed before the consequent. Half the time the English will give the consequent first, followed by its antecedent. Furthermore, the same operator is often capable of producing conditionals with the components in either order, as 'if' does in the two statements

If you touch me, I'll scream.

I'll scream if you touch me.

For this reason it is not sufficient to give a list of English conditional operators; it is necessary to indicate the forms of statement in which the antecedent is given first, and the forms of statement in which it is given last.

Structures in which the antecedent occurs first: $(A \supset B)$

> If ... then ... (If A then B)
> If ... , ... (If A, B)
> Given that ... it follows that ... (Given that A it follows that B)
> Not ... unless ... (Not A unless B)
> In case ... , ... (In case A, B)†
> Given that ... , ... (Given that A, B)
> Insofar as ... , ... (Insofar as A, B)
> ... implies ... (A implies B)*
> ... leads to ... (A leads to B)*
> Whenever ... , ... (Whenever A, B)
> ... only if ... (A only if B)
> ... is a sufficient condition for ... (A is a sufficient condition for B)
> ... means that ... (A means that B)
> So long as ... , ... (So long as A, B)

Structures in which the consequent occurs first: $(B \supset A)$

> ... if ... (A if B)
> ... in case ... (A in case B)†
> Unless ... , not ... (Unless A, not B)
> ... whenever ... (A whenever B)
> ... insofar as ... (A insofar as B)
> ... follows from (A follows from B)*
> ... is implied by ... (A is implied by B)*
> ... is a necessary condition for ... (A is a necessary condition for B)
> Only if ... , ... (Only if A, B)
> ... provided that ... (A provided that B)
> ... so long as ... (A so long as B)

The following expressions are operators which, in English, always come directly *before* the *antecedent* of the conditional, regardless of the order of the components:

> if (but not 'only if' and not 'even if')
> given that
> in case
> whenever
> provided that

The operator 'only if' always comes directly before the *consequent* of the conditional, regardless of the order of the components.

The expression 'even if' is *not* a conditional operator. Its treatment will be explained later on.

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*Cf. Appendix A and Appendix C.
†Cf. Appendix F.

The bracketing auxiliary 'then' (and 'it follows that') always comes immediately after the antecedent and immediately before the consequent.

The expression 'then if' never occurs except in the middle of a conditional whose consequent is another conditional, as in the statement

> If today is Sunday, **then if** we go to the fair then we can watch the balloon go up.

The antecedent of this statement is 'today is Sunday' and the consequent is 'if we go to the fair then we can watch the balloon go up'. This statement will translate as

$$S \supset (F \supset W)$$

And it may be taken as an ironclad rule of thumb that the expression 'then if' will always translate as 'horseshoe pren', as in the emphasized portion of the above translation: 'If S **then if** F then W'.

Exercises I

Give the logical translation of each of the following.

☐ 1. Julia won't scream unless Percy kicks her.
   2. Julia will scream only if Percy kicks her.
   3. It will snow tonight, provided that it gets cold enough.
☐ 4. Whenever a candle is left in the sun, it melts and runs all over.
   5. The candle won't melt, provided that you don't leave it in the sun or do something else foolish.
   6. If Southwestern loses and doesn't get to be in the playoffs, Percy will cry and pout.
☐ 7. If the engine is all right, then if we aren't out of gas, then something must be wrong with the battery.
   8. It will snow tomorrow, and if the kumquats are in blossom the crop will be ruined.
   9. If it snows tomorrow and the kumquats are in blossom, the crop will be ruined.
  10. If it snows or freezes tomorrow, then if the kumquats are in blossom and are unprotected, then the crop will be ruined unless a miracle occurs.
☐ 11. Given that it freezes only if it snows, and that it won't freeze unless the sky is clear, and that a clear sky is a sufficient condition for its not snowing, it follows that the crop is safe if the locusts don't get to it.
  12. If, whenever the sky is cloudy it either rains or snows, and whenever the sky is not cloudy it freezes, then given that freezing will ruin the kumquats and that snow will ruin the mulberries, it follows that if you planted mulberries instead of kumquats, and the sky is clear, you have nothing to worry about unless freezing ruins mulberries.

Exercises II

Supposing that A and B are both true statements, and that X and Y are both false statements, determine which of the following compound statements are true.

□ 1. $X \supset A$

2. $A \supset X$

3. $Y \supset Y$

□ 4. $B \supset B$

5. $X \supset (X \supset Y)$

6. $(X \supset X) \supset Y$

□ 7. $(A \supset X) \supset Y$

8. $(X \supset A) \supset Y$

9. $A \supset (B \supset Y)$

□ 10. $(X \supset A) \supset (B \supset Y)$

11. $(A \supset B) \supset (\sim A \supset \sim B)$

12. $(X \supset A) \supset (\sim X \supset \sim A)$

□ 13. $(X \supset \sim Y) \supset (\sim X \supset Y)$

14. $((A \cdot X) \supset Y) \supset (B \supset X)$

15. $((A \cdot X) \supset B) \supset (A \supset (B \supset X))$

16. $((X \supset Y) \supset X) \supset X$

§4.3 Subjunctive Conditionals

It was mentioned before that not all conditionals are truth-dependent compounds. Some examples of conditionals which are not truth-dependent are:

> If I were you, I would take poison.
> If the South had won the Civil War, it would still have slavery.
> If Jupiter were a star, there would be only eight planets.
> You wouldn't dare insult me sir, if Jack were only here.
> The poison wouldn't have killed him if he hadn't taken such a large dose.
> If the general had been less cautious, he would have been more successful.

The common feature of such conditionals is that they are phrased in what grammarians call the "subjunctive mood." Their most usual identifying feature is the auxiliary 'would' attached to the main verb in the consequent. Subjunctive conditionals are also called "counterfactual conditionals" and "contrary-to-fact conditionals"; a more appropriate label, however, would be "speculative conditionals," since the subjunctive mood is one of speculation, and this *need* not involve any contrariness-to-fact.

Subjunctive conditionals are called "conditionals" because they are couched in sentences whose clauses are connected by conditional operators.

They are not truth-dependent compounds, since in the first place it is dubious whether they are compound *statements* at all. They are not compound statements unless their clauses express propositions, which are either true or false, and this does not seem to be the case. What proposition, for example, is expressed by a phrase such as 'I were you', or 'the general had been less cautious', or 'I would take poison'? Unlike their indicative counterparts 'I am you', 'the general was less cautious', and 'I will take poison', which have fairly straightforward truth-conditions, there seems to be no way to say what it would be like for 'I were you' to be true, *or* to be false.* But if, as this would indicate, the component clauses do not express propositions, then the subjunctive conditional is not a compound statement at all.

In the second place, when the truth or falsity of subjunctive conditionals is debated, the truth-value of the indicative counterparts of their clauses is not regarded as a relevant consideration. They are debated on quite different grounds than that, which is a further indication that they are not compound statements. For these reasons it is standard practice, in elementary logic at any rate, to treat subjunctive conditionals as *simple* statements and let matters go at that.

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*Some logical theoreticians have suggested regarding the clauses of subjunctive conditionals as expressing the same propositions as their indicative counterparts, thereby equating 'If I were in Las Vegas, I would be having a good time' with 'If I am in Las Vegas, I am having a good time'. But this is unsatisfactory, for reasons that should be obvious. The latter is equivalent to 'If I'm not having a good time, then I'm not in Las Vegas', which someone could confirm, say, by ascertaining that I am not having a good time (in fact, I am in bed with a toothache), and then proceeding to discover that, sure enough, I am not in Las Vegas but in Grand Rapids. However, it is clear that these results (my *in fact* being in Grand Rapids and *in fact* being miserable) are wholly irrelevant to the question of what my state *would be* if I *were* somewhere else. Furthermore, since subjunctive conditionals are, as often as not, put forth with the full knowledge that the indicative counterparts of their clauses are false (so that they really *are* counterfactual conditionals), it would follow by the material interpretation that every such conditional is *true*, including such anomalies as 'If the moon were square, there would be no tides', which most of us would regard as false despite the fact that 'The moon *is* square $\supset$ there *are* no tides' turns out to be true because of the mutual falsity of its components.

BICONDITIONALS; TRUTH-TABLES;

LOGICAL STATUS

§5.1 Biconditionals

One further type of compound statement is less common than those we have looked at so far. It is called the **biconditional**. A biconditional is a conditional which runs both ways. In ordinary discourse, biconditionals are frequently expressed as the conjunction of two conditionals, one of which is the reverse of the other. For example:

1. If that's poison, it will kill mice; and if it will kill mice, it's poison.
2. If it's raining, then the streets are wet; and if the streets are wet, it's raining.
3. Whenever I'm drinking beer, I'm having a good time; and whenever I'm having a good time, I'm drinking beer.

As often as not, one of the two conditionals will be replaced by its contrapositive. Thus, instead of (1), we might also say

4. If that's poison, it will kill mice; and if it's not poison, it won't.

Other examples of this are:

5. If litmus comes in contact with acid, it turns red; and if litmus doesn't come in contact with acid, it doesn't turn red.
6. The baby won't stop crying unless you feed him; but if you feed him, he'll stop.
7. If Jones is dead, then he isn't breathing; but if he isn't dead, then he is breathing.

When a biconditional is phrased in either of these two ways, it can easily be translated into the symbolic language with the operators already at our disposal. For example, (1) above might be translated as

1a. $(P \supset K) \cdot (K \supset P)$

Similarly, (5) might be translated as

5a. $(A \supset R) \cdot (\sim A \supset \sim R)$

For that matter, *all* biconditionals might be translated as the conjunction of two conditionals if we so desired. But this leads to fairly cumbersome formulae, and so for purposes of brevity we generally employ a biconditional operator.

In logic there is only one biconditional operator, the symbol '≡', called **triple bar**. When the triple bar is placed between two statements, abbreviated or otherwise, the resulting compound is a **material biconditional** whose components are the statements on either side of the triple bar. A material biconditional is a compound statement which is true if its components (they are called its **sides**.) both have the same truth-value, and is false if they have different truth-values.* The truth-table definition of the triple bar is

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Like the dot, wedge, tilde, and horseshoe, the triple bar is a pure truth-functional connective which has no exact synonym in English. When reading the symbolic language aloud, we customarily read the '≡', as "triple bar" or "if and only if."

When a biconditional is expressed as the conjunction of two conditionals, as in examples (1)–(7), the procedure for translating it by means of the triple bar is the following. Select *one* of the two conditionals (normally the first one, but it doesn't matter which) and use the components of that conditional as the sides of the biconditional. For example, (1) would be translated, using the triple bar, either as

1b. That's poison ≡ it will kill mice

or as

1c. That will kill mice ≡ it's poison

Similarly, (5) will translate in either of the following two ways:

5b. Litmus comes into contact with acid ≡ it turns red

5c. Litmus doesn't come in contact with acid ≡ it doesn't turn red

Exercises I

Translate the remaining examples (2, 3, 4, 6, and 7) first as the conjunction of two conditionals and then as a biconditional using the triple bar.

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*Material biconditionals are sometimes called "statements of material equivalence," and practitioners of logic sometimes say that the sides of a material biconditional are "materially equivalent" to each other. "Materially equivalent" means "equivalent in truth-value" and nothing more.

TRANSLATION AID

The following English expressions are biconditional operators.

> . . . if and only if . . .
> . . . is equivalent to . . .
> . . . is a necessary and sufficient condition for . . .
> . . . just in case . . .*
> . . . just if . . .

Exercises II

Translate the following.

1. If Harvey is a bachelor if and only if he is not married, then he has a wife only if he is not a bachelor.

□ 2. If Harvey has a wife only if he is married, and he is a bachelor if and only if he is not married, then he is not a bachelor if he has a wife.

3. Either Billy didn't hit Jenny or they are over at the neighbor's house, inasmuch as Billy's hitting her is a necessary and sufficient condition for her screaming, and I didn't hear any screams.

4. I will swat Billy a good one, just in case Jenny is bruised if he hit her.

§5.2 Truth-Tables

So far, truth-tables have been used only for the purpose of defining the truth-functional operators of the symbolic language. But this is not their only use, nor even their most important one. The truth-table is also an instrument for logical evaluation.

Every statement which is built up out of simple statements and truth-functional operators, no matter how complex it may be, is a truth-function of the simple statements in it. The truth-values of its simple statements will ultimately determine its truth or falsity; hence, the truth-conditions for the entire compound are to be specified in terms of the (possible) truth-values of its simple statements. One of the most convenient ways of establishing the truth-conditions of a compound statement is by building a properly constructed truth-table for that statement. For no matter how complex a statement may be, a truth-table will show, clearly and unambiguously, the conditions which would make it true and those which would make it false. This is no small advantage, since unaided intelligence, no matter how great, will normally boggle if asked to specify the truth-conditions for something such as, say,

$$\sim(C \supset (D \equiv (C \lor \sim D))) \supset (\sim(C \cdot (E \lor (D \cdot C))) \equiv E)$$

*But see Appendix F.

To refresh our memory, a truth-table may be given the following precise definition:

> **Truth-Table.** A truth-table is a diagram which displays all possible combinations of truth-values of a given collection of statements, and which shows how each such combination affects the truth-value of some compound which is a truth-function of those statements.

Thus, a truth-table has two parts: the part which displays the possible combinations of truth-values, and the part which shows the effects of each. Learning how to use truth-tables involves learning how to construct both parts correctly.

Let us begin with an example. Consider the statement

1. If Clyde isn't invited to the party, then Harvey won't come.

translated as

1a. $\sim C \supset \sim H$

The complete truth-table for this statement is:

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Briefly, what this truth-table tells us is that '$\sim C \supset \sim H$' is false when 'C' is false and 'H' is true, but is true otherwise. The columns to the left of the double vertical line—the **base columns**—display all possible combinations of truth-values of the simple statements occurring in '$\sim C \supset \sim H$' (namely, 'C' and 'H'). Reading the table across from left to right, and ignoring everything between the base columns and the final column to the right, the top row below the horizontal line tells us that when 'C' and 'H' are both true, then '$\sim C \supset \sim H$' is true; the second row down tells us that when 'C' is true and 'H' is false, then '$\sim C \supset \sim H$' is true; and so on.

Let us now look at how this truth-table was constructed. First, a horizontal line (sufficiently long to accommodate the truth-table) is drawn, and the compound under investigation is written above the line at the far right, thus:

$$\sim C \supset \sim H$$

Next, this formula is investigated to see if either of its components is itself compound. In fact, both of them are, so these two formulae are written above

the line and to the left, with appropriate separators, thus:

	$\sim C$	$\sim H$	$\sim C \supset \sim H$

Next, these formulae are investigated to see if either of them has components which are themselves compound. Neither of them has, so the simple statements occurring in the original are written above the line and to the left, with a double separator, thus:

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$

Next, base columns are constructed beneath the simple statements (the general rules for doing this are given later), thus:

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$
T	T			
T	F			
F	T			
F	F			

At this point we are ready to begin constructing **derived columns**. We construct a derived column downward, one step at a time, by looking at the column(s) for the *component(s)* of the formula whose column we are deriving, and deciding at each row how *that* combination of truth-values affects the truth-value of the formula whose column we are deriving. This decision is made on the basis of the truth-table definitions of the various operators, which should be committed to memory. In deriving the column for '$\sim C$' we look at the column for its component—the column for 'C'. The formula '$\sim C$' is a negation; from the truth-table definition we know that a negation is true when its negate is false and false when its negate is true. Thus at the first row, where 'C' has the value T, we write the value F in the column for '$\sim C$'. We proceed on down in this fashion until the entire column is completed, thus:

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$
T	T	F		
T	F	F		
F	T	T		
F	T	T		

Next, a column is derived for '$\sim H$':

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

And finally we are able to derive a column for '$\sim C \supset \sim H$'. Looking at the columns for its components (the columns for '$\sim C$' and '$\sim H$') and recalling the definition of the horseshoe, we write a T in the first row, a T in the second row, an F in the third row (since a conditional with a true antecedent and a false consequent is false), and a T in the fourth row:

C	H	$\sim C$	$\sim H$	$\sim C \supset \sim H$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

With this account in mind, let us look at some general rules for constructing a truth-table. The proper array of formulae above the line is in order of increasing dominance from left to right. That is, at the far right comes the compound statement under consideration. To its left come (in no particular order) its components which are not simple statements (if any). To their left come their components which are not simple statements, if any; and so on. Finally, at the far left, come all of the simple statements in the original compound. It is customary to arrange them alphabetically from left to right, and this custom should be adhered to.

There is one cardinal rule in constructing derived columns, which should be obvious but which many beginners in logic nevertheless overlook: *never attempt to derive a column for a formula unless the table already contains columns for each of that formula's components.*

§5.3   Constructing Base Columns

The base columns for a set of *two* statements (such as those in the above example) can be seen intuitively to cover all possible combinations of truth-values of the two. But as the number of statements increases, this intuitive clarity tends to dim. Thus, it is helpful to have a rule to follow which will assure that no possibilities are left out of the base columns for larger collections.

The number of rows in a truth-table *doubles* every time another simple statement is added to the collection. A truth-table for one statement has

two rows; for two statements it has four rows; for three statements, eight rows; for four statements, sixteen rows; and so on. This is sometimes expressed by saying: For a collection of n statements, the truth-table will contain 2^n (2 to the nth power) rows. This "doubling" feature is utilized in the rules for base-column building:

A. Under the letter furthest to the *right*, construct a column of alternating T's and F's (T F T F T F . . .). Base columns always begin with T's, never F's.
B. Working to the left, the clustering of T's and F's *doubles* for each succeeding letter. That is:
 i. Under the letter second from the right, construct a column of alternating *pairs* of T's and F's (T T F F T T F F . . .).
 ii. Under the letter third from the right, construct a column of alternating *quartets* of T's and F's (T T T T F F F F . . .).
 iii. And so on.
C. The base columns are completed when the column furthest to the *left* has one cluster of T's and one cluster of F's. At that point, the top row will be all T's, the bottom row will be all F's, and all other possibilities will occur in between.

As an illustration, here is a complete truth-table for the formula:

$$((A \supset B) \lor \sim(A \cdot C)) \supset B$$

A	B	C	$(A \supset B)$	$(A \cdot C)$	$\sim(A \cdot C)$	$(A \supset B) \lor$ $\sim(A \cdot C)$	$((A \supset B) \lor$ $\sim(A \cdot C)) \supset B$
T	T	T	T	T	F	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	F	T
T	F	F	F	F	T	T	F
F	T	T	T	F	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	F	T	T	F
F	F	F	T	F	T	T	F

§5.4 Tautologies, Contradictions, and Contingencies

Some compound statements are such that they are true for all possible combinations of truth-values of their components; they are "true no matter what the world is like." A statement of this type is called a **tautology** and is said to be **tautologous**. A simple example would be the statement "Either it is raining or it is not raining" $(R \lor \sim R)$. It is shown to be a tautology by the following truth-table:

R	$\sim R$	$(R \lor \sim R)$
T	F	T
F	T	T

Notice that the statement $(R \lor \sim R)$ has only T's in the column beneath it. This is the defining mark of a tautology: its column in the truth-table contains all T's and no F's. Not all tautologies are as simple as this one; for example, the statement $(A \supset B) \supset \sim(A \cdot \sim B)$ is also a tautology. But no matter how complex a statement you are faced with, it is always an easy matter to discover whether or not it is a tautology. Simply construct a truth-table for it, then inspect its column to see whether it contains any F's. If there are no F's, then the statement is true under all possible conditions and is a tautology. But if the column contains one or more F's, then the statement could be false and is not a tautology.

Likewise, some compound statements are such that they are *false* for all possible combinations of truth-values of their components; they are *false* no matter what the world is like. A statement of this type is called a **contradiction** and is said to be **contradictory**. A simple example would be 'It is raining and it is not raining' $(R \cdot \sim R)$. As before, this can be shown to be a contradiction by a truth-table:

R	$\sim R$	$(R \cdot \sim R)$
T	F	F
F	T	F

The defining mark of a contradiction is that it has only F's in its truth-table column. Just as there are very complex tautologies, so there are very complex contradictions. But again, it is always possible to tell whether a statement is a contradiction by constructing a truth-table for it and inspecting its column for T's. If it contains only F's, then it is a contradiction. But if it contains one or more T's, it could be true, and it is not a contradiction.

A statement which is neither a tautology nor a contradiction is called a **contingency** and is said to be **contingent**. A contingency has at least one T *and* at least one F in its truth-table column, and so it is always possible to tell whether a statement is a contingency by constructing a truth-table for it and checking to see whether it has a T *and* an F in its truth-table column. If it has, it is a contingency; if not, it is either a tautology or a contradiction.

§5.5 Logical Status

Every statement is either a tautology, a contradiction, or a contingency. To say *which* of these three a given statement is, is to give the statement's **logical status**. If you are given a statement and asked to determine its logical status, that means you are to determine whether it is a tautology or a contradiction or a contingency. The usual way of doing this is by a truth-table.

Exercises III

Construct a truth-table for each of the following statements in order to ascertain its logical status.

☐ 1. $A \supset \sim A$

2. $\sim A \supset A$

3. $(\sim A \supset A) \supset A$

☐ 4. $(A \supset \sim A) \supset \sim A$

5. $\sim A \supset (A \supset B)$

6. $\sim A \supset (A \supset \sim B)$

☐ 7. $A \supset (B \supset A)$

8. $A \supset (\sim B \supset A)$

9. $((A \supset B) \supset A) \supset A$

☐ 10. $((\sim A \supset \sim B) \supset A) \supset A$

11. $(A \supset B) \cdot \sim (B \lor \sim A)$

12. $(A \supset B) \supset ((A \supset B) \lor C)$

☐ 13. $\sim (A \lor \sim A)$

14. $(\sim (A \supset A) \lor (\sim B \cdot \sim \sim B)) \lor C$

15. $A \supset (A \supset A)$

☐ 16. $A \supset ((A \supset A) \supset A)$

17. $(A \cdot C) \supset (\sim D \lor (B \supset A))$

18. $(A \supset A) \supset \sim (B \supset B)$

☐ 19. $\sim (B \supset B) \supset (A \cdot \sim A)$

20. $(((((A \supset B) \supset A) \supset A) \supset A) \supset A) \supset A$

chapter 6

A LOGIC–ENGLISH TRANSLATION GUIDE

This chapter will bring together the previously given Translation Aids and also provide additional information on translating certain expressions.*

§6.1 Conjunctive Operators

and
but
although
however
... also ...
whereas
both ... and ...
but even so
after all
for
nevertheless
still (*except in the sense of* 'any more')
besides
even though (*but not* 'even if'; *see* §6.5)
not only ... but also ...
in spite of the fact that
plus the fact that
inasmuch as (*but not* 'insofar as'; *see* §6.3)
while (*in the sense of* 'although', *but not* 'during which time')
since (*in the sense of* 'whereas', *but not* 'after')
as (*in the sense of* 'whereas', *not* 'at the same time as')

Many of the words and expressions in English which serve as conjunctive

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*An index of logical operators may be found at the back of the book.

operators sometimes play another role as temporal indicators—to indicate the time-relationship between the events mentioned in the conjuncts. This split personality belongs to the following connectives from the above list:

> still
> while
> since
> as
> and (*when used in the sense of* 'and then')

When 'still' is used in its temporal sense (as opposed to its straightforwardly conjunctive sense), it occurs as an auxiliary to the verb in one of the clauses rather than as a device to connect the clauses. For example:

> *Temporal:*   Harvey has been gone twenty years but she still remembers him.
> *Conjunctive:* Percy is a good boy; still, he sometimes gets into mischief.

Whenever 'still' is accompanied by another conjunctive operator in the same statement, the *other* operator is the one which connects the components; thus

> 1. Even though Clyde has been warned, he still drives recklessly.

will translate as

> 1a. Clyde has been warned · he still drives recklessly

rather than as

> 1b. Even though Clyde has been warned · he drives recklessly

which makes no sense at all.

Unfortunately, the other split-personality conjunctive operators give no such grammatical clues as to which of their personalities is showing; we must determine this, if at all, by understanding the particular statement in which the connective occurs. But here are some examples.

'while' 2. While Edna tended the campfire, Clyde went in search of water.
> (*Temporal:* 'while' means 'at the same time as'.)

> 3. While New York is a nice place to visit, I wouldn't want to live there.
> (*Conjunctive:* 'while' means 'although'.)

'since' 4. Since Clyde became famous, he has forgotten his old friends.
> (*Temporal:* 'since' means 'after' or 'subsequently'.)

> 5. Since Harvey had the best qualifications, he got the job.
> (*Conjunctive:* 'since' means 'whereas'.)

'as'   6. Clyde came down the stairs as Edna entered the house.
> (*Temporal:* 'as' means 'at the same time'.)

> 7. They attend the theater daily as they do not care for television.
> (*Conjunctive:* 'as' means 'inasmuch as'.)

'and'   8. Harvey got out of bed and took a shower.
> (*Temporal:* 'and' means 'and afterwards'.)

> 9. Clyde hates Thelma and Thelma hates Clyde.
> (*Conjunctive:* 'and' means 'and', period.)

When a split-personality conjunctive operator has its temporal sense, the resulting compound is not truth-functional and the operator cannot simply be translated as a dot. However, such compounds are truth-dependent. Their method of treatment is discussed in Appendix C.

It may be taken as a convention that, if there is no way to tell whether a split-personality operator has its temporal sense, it is to be treated as straightforwardly conjunctive and translated as a dot.

§6.2 Disjunctive Operators

> or
> either . . . or . . .
> or else
> or, alternatively
> otherwise
> with the alternative that
> unless

The word 'or', besides being a disjunctive operator, is often used as a synonym for 'that is to say'. For instance, the naturalist who says

> 1. This creature is a *bufo vulgaris* or common toad.

is not claiming that either it is the one or it is the other; rather, he is claiming that it is the one, *that is to say*, the other, since the two are different expressions for the same species of creature. Though not strictly proper to translate this 'or' as a wedge, it is harmless to do so since the resulting disjunction will always have the same truth-value as the original statement.

§6.3 Conditional Operators

Forms in which the antecedent comes first: (. . .) ⊃ (– – –)

> if . . . then – – –
> if . . . , – – –
> given that . . . it follows that – – –
> given that . . . , – – –
> not . . . unless – – –
> in case . . . , – – –*

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*Cf. Appendix F.

insofar as . . . , – – –
so long as . . . , – – –
. . . implies – – –†
. . . leads to – – –†
. . . only if – – –
whenever . . . , – – –
. . . is a sufficient condition for – – –
. . . means that – – –
to the extent that . . . , – – –

Forms in which the consequent comes first: (– – –) ⊃ (. . .)

– – – if . . .
– – – in case . . .*
unless – – – , not . . .
– – – whenever . . .
– – – insofar as . . .
– – – so long as . . .
– – – follows from . . .†
– – – is implied by . . .†
– – – is a necessary condition for . . .
only if – – – , . . .
– – – provided that . . .
– – – to the extent that . . .

The inclusion of 'not . . . unless . . .' as a conditional operator may cause confusion, since 'not' has already been classified as a negative operator and 'unless' has already been classified as a disjunctive operator. By the latter,

    1. not A unless B

should apparently be translated as

    2. $\sim A \lor B$

whereas, if 'not . . . unless . . .' is a conditional operator, then the same statement should apparently be translated as

    3. $A \supset B$

—and how is a student to know which translation to use? The answer is: use either one, since they are logically equivalent to each other. If the 'not' and the 'unless' in (1) are regarded as separate operators, the appropriate translation is (2). But if they are regarded as portions of the same operator (the operator 'not . . . unless . . .'), the appropriate translation is (3). From a logical point of view, either translation will be correct, since (2) and (3) have exactly the same truth-conditions and are therefore equivalent.

It is important that the 'not' be regarded *either* as a negative operator *or* as part of a conditional operator *but not both*. Forcing it to do double duty

*Cf. Appendix F.
†Cf. Appendix A and Appendix C.

invariably leads to mistranslations; for example, to the mistranslation of (1) as

    4. $\sim A \supset B$

In colloquial English the expressions 'inasmuch as' and 'insofar as' are often used interchangeably. But apart from colloquialisms, the two expressions differ not only in their meaning, but in their logical force. 'Inasmuch as' is a *conjunctive* operator, whereas 'insofar as' is a *conditional* operator. The person who asserts a compound formed with the operator 'inasmuch as' is claiming that both components are true; the person who asserts a compound formed with the operator 'insofar as' is only claiming that, to the extent that the one component is true, so is the other—that is, if the one is true then the other is also. For example, if a detective says

    5. Smith had no motive for the robbery, inasmuch as he is wealthy.

his assistant might, quite properly, contradict him by saying "You're wrong about that; Smith doesn't have a dime to his name." And by recognizing that this reply *does* contradict (5), we see that part of what (5) asserts is that Smith is wealthy. However, the assistant might equally well have contradicted (5) by saying "You're wrong about that; Smith had an excellent motive." Clearly, in asserting (5), the detective is claiming both that Smith is wealthy and that he had no motive for the robbery. Contrast this with

    6. Smith had no motive for the robbery, insofar as he is wealthy.

Here, if the assistant replied "You're wrong; he had an excellent motive," the detective could properly reply, "That doesn't mean I'm wrong; it means Smith isn't really wealthy." For in asserting (6) the detective has not asserted flatly that Smith has no motive; rather he has asserted conditionally that Smith, *insofar as he is wealthy*, has no motive. Similar differences exist between the members of the following pairs of statements:

    7. Inasmuch as Jones is dead, his wife is entitled to the insurance.
    8. Insofar as Jones is dead, his wife is entitled to the insurance.
    9. Inasmuch as the plane was on time, Harvey will make it to the meeting.
    10. Insofar as the plane was on time, Harvey will make it to the meeting.
    11. Inasmuch as Lincoln was responsible for the Civil War, he was responsible for the excesses of the Reconstruction.
    12. Insofar as Lincoln was responsible for the Civil War, he was responsible for the excesses of the Reconstruction.

In each case the first member of the pair is a conjunction, and the second is a conditional.

It is important to remember that the operator 'if' always precedes the *antecedent*, while the operator 'only if' always precedes the *consequent*. For example,

    13. A if B

means the same as 'If B then A', and translates as $B \supset A$, whereas

14. *A* only if *B*

means the same as 'Not *A* unless *B*', and translates as $A \supset B$.

A curiosity of the English language is that 'and' will function as a conditional operator, rather than a conjunctive one, when the clause preceding it is phrased in the Imperative mood and the clause following it is Declarative. For example,

15. Touch me and I'll scream!

is an alternative way of expressing the straightforward conditional

16. If you touch me, I'll scream!

Other examples of the same thing are:

17. Do that once more and you will get a spanking.
18. Ask her and she'll marry you.
19. Drink from that glass and you will die.
20. Bring me my slippers and I'll give you a nickel.

However, when the clause following it is also in the Imperative, 'and' once again becomes conjunctive. But the result will be a conjunction of imperatives rather than of propositions, and so will fall outside the scope of logic. For example,

21. Bring me my slippers and give me a nickel.
22. Touch me and scream.
23. Drink from that glass and tell me how you like it.
24. Love her and leave her.

are not conditionals, but conjunctive imperatives.

§6.4 Biconditional Operators

The biconditional operators likely to be encountered in English are:

> if and only if
> if but only if
> is equivalent to
> is a necessary and sufficient condition for
> just in case*
> just if
> just insofar as
> just to the extent that

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*Cf. Appendix F.

§6.5 Miscellany

'even . . .'

In colloquial English the two expressions 'even though' and 'even if' are frequently used interchangeably. Nevertheless (just as with 'inasmuch' and 'insofar') the expressions differ not only in meaning but also in logical force. 'Even though' is a conjunctive operator, and compounds formed with its help are to be translated with the dot. But 'even if' is not a compounding operator at all. It functions rather as a disclaimer, and its force is that the truth or falsity of what follows it doesn't matter to what precedes it. Similar disclaimer-expressions are 'whether or not', 'regardless of (whether)', and 'irrespective of (whether)'.* Some examples of these are:

1. I won't tell the secrets, even if you have decided to torture me.
2. It rained yesterday, whether or not you noticed it
3. My client is innocent, regardless of what the police think.
4. I shall be there irrespective of whether it rains.

The logical force of such statements is simply to assert that their first component is true. The segment following the disclaimer-expression is, logically, a decoration added on for emphasis. Let us call the other portion of the statement the "main component." Then this point may be made by saying that the truth-conditions for the entire statement are the same as the truth-conditions for the main component. For example, if I don't tell the secrets, then (1) is true, and if I do tell the secrets, then (1) is false. The question of torture is irrelevant, for, decision to torture or not, if I tell the secrets then (1) is false. Similarly, (3) is true if the client is innocent and is false if the client is not innocent. The opinions of the police are irrelevant to the truth or falsity of (3). These may be contrasted with the genuine compounds (conjunctions):

5. I won't tell the secrets, even though you have decided to torture me.
6. It rained yesterday, even though you didn't notice it.
7. My client is innocent, even though the police think he's guilty.
8. I shall be there, even though it's going to rain.

The proper logical treatment for statements containing disclaimer-expressions is to *discard* the disclaimer-expression, together with what follows it, and to translate the statement as its main component. This is appropriate because the disclaimer-expression and what follows it do nothing more than provide emphasis to the main component, without in any way affecting the

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*However, the two expressions 'regardless of the fact that' and 'irrespective of the fact that' are conjunctive operators. Here, the difference between 'whether' and 'the fact that' is the difference between merely supplying emphasis and supplying an additional conjunct.

truth-value of what is stated. Thus, the statement (1) above will abbreviate
as

    1a. $\sim S$ even if T

and its proper translation will be

    1b. $\sim S$

'neither ... nor'

The expression 'neither ... nor ...' translates as the denial of a disjunc-
tion, not as the disjunction of two denials; an alternative is to translate it
as the conjunction of two denials, but not as the denial of a conjunction.
For example,

    9. Neither Harvey nor Clyde was invited to the party.

paraphrases to

    9a. Neither Harvey was invited to the party nor Clyde was invited to the
        party.

which abbreviates as

    9b. Neither H nor C

which may be translated, with equal correctness, either as

    9c. $\sim(H \vee C)$

or as

    9d. $(\sim H \cdot \sim C)$

'not both ... and ...'

The expression 'not both ... and ...' translates as the denial of a conjunc-
tion, not as the conjunction of two denials; an alternative is to translate it
as the disjunction of two denials, but not as the denial of a disjunction. For
example,

    10. Not both Edna and Thelma will be selected Queen of the May.

paraphrases to

    10a. It's false that both Edna will be selected Queen of the May and Thelma
         will be selected Queen of the May.

which abbreviates as

    10b. It's false that both E and T

which may be translated, with equal correctness, either as

    10c. $\sim(E \cdot T)$

or as

    10d. $(\sim E \vee \sim T)$

'instead of'; 'rather than'; 'without'

The operators 'instead of', 'rather than', and 'without' should always be
paraphrased as 'but not'. For example, the statement

    11. They went to the movies instead of the reception.

paraphrases as

    11a. They went to the movies but not the reception.
    11b. They went to the movies but they did not go to the reception.
    11c. They went to the movies but it's not the case that they went to the recep-
        tion.

which abbreviates to

    11d. M but it's not the case that R

which translates as

    11e. $M \cdot \sim R$

Similarly, the statement

    12. Rather than to the reception, they went to the movies.

paraphrases as

    12a. They went to the movies rather than to the reception.
    12b. They went to the movies but not to the reception.

and

    13. They entered without paying.

becomes

    13a. They entered but they didn't pay.

and so on.

Exercises I

Some of the following can be translated into correctly punctuated formulae and
some cannot. Translate those which can be, and explain what is the matter with
each of the others.

    1. Either P or if Q then R
    2. If P then either Q or R
☐ **3.** Either if P or Q then R
    4. If P then both Q and R
    5. Either if P then Q or R
☐ **6.** If either P or Q then R
    7. Both P and if Q then R
    8. Neither both P nor Q and R
    9. If either P then Q or R

□ **10.** If P then neither Q nor R

11. Neither P nor if Q then R

12. Both if P then Q and R

□ **13.** Neither if P nor Q then R

14. If both P and Q then R

15. If neither P then Q nor R

□ **16.** If neither P nor Q then R

Exercises II

Translate the following, paying special attention to punctuation.

1. Either P without Q or R

□ **2.** P instead of either Q or R

3. Neither P instead of Q nor R

4. Not both P and Q unless either R or S

□ **5.** If P rather than both Q and R then S

6. If either P rather than Q or R then S

□ **7.** If either P rather than either Q or R or S then T or U

8. Either if P rather than either Q or R then T or U

9. Neither both either P or Q and R nor S

□ **10.** Either both neither P nor Q and R or S

11. Both neither either P or Q nor R and S

12. Not both either P if Q or R and S

□ **13.** If P then Q if R

14. If P if Q then R

15. If if P then Q then R

□ **16.** Neither if both either neither P without Q nor R or S and T then U nor W

17. Both if both neither if either both P if Q and R or Z then neither S nor T nor W and X then either both P instead of Q and R but not S or W and Z

chapter 7

CONDENSED TRUTH-TABLES; TRUTH-FUNCTIONAL EQUIVALENCE

§7.1   Condensed Truth-Tables

Chapter 5 explained how to construct a complete truth-table for any compound statement. Once one has mastered and understood the technique for constructing complete truth-tables, it is generally more convenient to employ what are called **condensed** truth-tables.

The major difference in a condensed truth-table is that nothing is written above the line except the compound statement under consideration and the simple statements occurring in it. A complete truth-table includes not only the statement under consideration, but its components, and the components of its components, and so on down to the simple statements; and the various columns are constructed beneath the appropriate formulae. A condensed truth-table has just as many columns as there would be in a complete truth-table, but they are constructed beneath the appropriate *operators*, within the formula under consideration. The *order* in which the columns are constructed is the same as in a complete truth-table: first for compounds of simple statements, then for compounds of those, and so on.

For example, suppose we wish to construct a condensed truth-table for the compound statement '$(\sim A \supset B) \lor (B \supset A)$'. We write all of the single letters, and the whole statement, above the line, and construct base columns:

A	B	$(\sim A \supset B) \lor (B \supset A)$
T	T	
T	F	
F	T	
F	F	

In a complete truth-table the subcomponent '$\sim A$' would be written out by itself above the line, and our next step would be to construct a column beneath it. In our condensed truth-table we do derive that column next, but

it is constructed directly beneath the tilde in '$\sim A$', thus:

A	B	$(\sim A \supset B) \lor (B \supset A)$
T	T	F
T	F	F
F	T	T
F	F	T

We then proceed as before, constructing columns for those formulae whose components already have columns in the table. Thus:

A	B	$(\sim A \supset B) \lor (B \supset A)$	
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Then:

A	B	$(\sim A \supset B) \lor (B \supset A)$		
T	T	F	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

And finally:

A	B	$(\sim A \supset B) \lor (B \supset A)$			
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	T	F
F	F	T	F	T	T

When a column has finally been derived beneath the dominant operator* in the original statement, the truth-table is finished. It is generally a good idea to draw a line around the column beneath the dominant operator to make it stand out as the column for the whole statement, so our completed truth-table looks like this:

A	B	$(\sim A \supset B) \lor (B \supset A)$			
T	T	F	T	T	T
T	F	F	T	T	T
F	T	T	T	T	F
F	F	T	F	T	T

*See §3.4

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§7.2 Truth-Functional Equivalence

Two statements are said to be **logically equivalent** if they are both true under exactly the same conditions and are both false under exactly the same conditions—that is, if they have identical truth-conditions. Truth-functional compounds which are logically equivalent are **truth-functionally equivalent.*** Thus, truth-functional equivalence is a species of logical equivalence. Whether or not two statements are truth-functionally equivalent may easily be determined by a truth-table, since it displays the entire set of truth-conditions for a compound statement.

To determine whether two statements are truth-functionally equivalent, first take all the letters (simple statements) from *both* of them and write these above the line; then write the two compounds which are being compared above the line. Construct a set of base columns beneath the single letters and finish out the truth-table for both of the compounds being compared. Next compare, row by row, the columns under the dominant operators for the two compounds. If they are exactly alike all the way down, then the two statements are truth-functionally equivalent. But if they differ anywhere—that is, if there are any rows where one of the columns has T and the other has F—then the two statements are *not* truth-functionally equivalent.

In our notation there is no separate symbol for logical equivalence. Since the full English phrasing " 'P' is logically equivalent to 'Q' " is long and cumbersome, we shall employ the abbreviation '*equiv*' between formulae without quotation marks. Thus, instead of

1. '$(A \cdot B)$' is logically equivalent to '$(B \cdot A)$'

we shall say

1a. $(A \cdot B)$ *equiv* $(B \cdot A)$

as an abbreviated version of the same thing. For example, the first truth-table below shows that

2. $(A \supset B)$ *equiv* $(\sim A \lor B)$

while the second one shows that

3. $(A \supset A)$ *equiv* $(((A \supset B) \supset A) \supset A)$

and the third one shows that '$A \supset B$' is *not* logically equivalent to '$B \supset A$'—that is, that

4. '$(A \supset B)$ *equiv* $(B \supset A)$' is false.

*There are complicated and logically trivial exceptions to this.

or, extending our abbreviation, that

4a. $(A \supset B)$ *not equiv* $(B \supset A)$

A	B	$(A \supset B)$	$(\sim A \vee B)$
T	T	T	F T
T	F	F	F F
F	T	T	T T
F	F	T	T T

The two circled columns are identical, row for row; hence the two statements are truth-functionally equivalent. Next,

A	B	$(A \supset A)$	$(((A \supset B) \supset A) \supset A)$
T	T	T	T T T
T	F	T	F T T
F	T	T	T F T
F	F	T	T F T

The two circled columns are identical, row for row; hence the two statements are truth-functionally equivalent. Finally,

A	B	$(A \supset B)$	$(B \supset A)$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

The two circled columns differ at the second row (and again at the third row); hence the two statements are not truth-functionally equivalent.

Exercises I

Determine which of the following pairs of statements are truth-functionally equivalent.

□ **1.** $\sim(A \cdot B)$, $(\sim A \cdot \sim B)$
 2. $\sim(A \supset B)$, $(A \cdot \sim B)$
 3. $(\sim A \vee A)$, $(\sim C \vee C)$
□ **4.** $(A \cdot \sim A)$, $(C \cdot \sim C)$
 5. $(A \cdot (B \vee C))$, $((A \vee B) \cdot (A \vee C))$
 6. $(A \vee (B \cdot C))$, $((\sim A \supset B) \cdot (\sim A \supset C))$
□ **7.** $\sim(A \vee B)$, $((A \supset B) \cdot \sim B)$
 8. $\sim(B \supset C)$, $(B \supset \sim C)$
 9. $(A \supset B)$, $(\sim B \supset \sim A)$
□ **10.** $(\sim A \supset (A \supset B))$, $(B \supset (A \supset A))$

 11. $((A \equiv B) \supset C)$, $(\sim C \supset ((A \supset B) \supset B))$
 12. $((A \supset A) \supset A)$, $(A \equiv (A \supset A))$

Exercises II

Give logical translations of the following.

□ **1.** If it's not the case that either it doesn't rain or Charlie stays home, then it rains only if Charlie stays home.
 2. If Clyde hates Thelma insofar as Thelma loves Harvey, then Thelma loves Harvey provided that Clyde hates her.
 3. If Percy didn't want to kick Julia unless he kicked her, then if he kicked her if he wanted to, then he didn't kick her just in case he didn't want to.
□ **4.** Either both if Lucinda kicked Percy then Percy kicked Julia and Evangeline fainted if Julia kicked Percy or Percy kicked Evangeline, if Lucinda hates Percy.
 5. Either if both either Harvey went to the mountains or to the beach and either both Edna went with him and Thelma is jealous or Clyde invited Thelma to the movies and she is happy then both either if Harvey hates Thelma then her heart will be broken or if Clyde loves Edna then Thelma's heart will be broken and Harvey didn't go to the mountains unless Edna went with him or else if if Harvey went to the beach then Thelma went with him then Clyde had to settle for Edna.
 6. If today is Monday and tomorrow is Saturday, then I'm Julius Caesar.
□ **7.** Either today is Monday and I have a test tomorrow or my calendar is wrong.
 8. Either today is Monday or my calendar is wrong and I don't know what day it is.
 9. Today is Friday, since yesterday was Wednesday.
□ **10.** Provided that yesterday was Wednesday, today is Friday.
 11. I have a test inasmuch as yesterday was Wednesday and today is Friday.
 12. Abner wants someone to take him to town, but Bill's car isn't working and Charlie is sick in bed.
□ **13.** If Charlie gets well or Bill fixes his car, then Abner will get to town after all.
 14. If Bill's car is beyond repair, then Abner won't get to town unless Charlie gets well.
 15. If Abner made it to town, that implies that either Charlie got well or Bill fixed his car.
□ **16.** If it freezes tonight, then if you don't have antifreeze in your car, then if you haven't drained your radiator, then if no damage occurs, then you are the luckiest man in town.
 17. If Charlie gets sick if he drinks too much, then given that he is healthy if and only if he is sober, he drinks too much if he gets sick.
 18. If Charlie isn't sick unless he's drunk, then his being either both well and sober or sick rather than drunk implies that his being sick is a necessary condition for his being drunk.

□ **19.** If either Charlie isn't in good health unless he's either sober or sleeping well or he has insomnia, then his being either both sick and drunk or in good health in case he's sober implies that his being sick is a sufficient condition for his either being drunk or not sleeping well.

20. If Bill spent the money on riotous living instead of for groceries, then if either the welfare check is late or his mother loses her job at the car-wash, then there won't be any groceries if Bill doesn't think of something pretty quick.

chapter 8

ARGUMENTS

§8.1 Validity

An argument is valid if there is no way for all of its premisses to be true and its conclusion false simultaneously. An argument may be valid either in virtue of its **form,*** or in virtue of the meanings of its premisses and conclusion. One which is valid in virtue of its form is said to be **formally valid.** One which is valid, but which is not valid in virtue of its form, is said to be **informally valid.**

Prior to the development of modern symbolic logic, many kinds of argument now recognized as formally valid were relegated to the other category. However, with the help of modern techniques and a more sophisticated conception of logical form, their formal validity can be, and has been, demonstrated. This does not mean that there is no such thing as an informally valid argument. There are many such, at any rate so far as we can tell at present, but their existence is a thorn in the side of formal logicians. An example of an informally valid argument would be

> Harvey drank six glasses of water.
> Thelma is Harvey's wife.
> _____
> Therefore, Thelma's husband drank several glasses of liquid.

The reason why informally valid arguments are troublesome is that there is no rigorous procedure for *proving* their validity. We must rely upon intuition instead, with all its notorious fallibilities. Thus one of the major goals of theoreticians in applied formal logic is to reduce, as far as possible, the class of informally valid arguments by showing, with the help of increasingly complex and sophisticated techniques, that such arguments are really valid in virtue of some logical form which reveals itself under detailed analysis.

Formal logic, the subject of this book, is not concerned with informally

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*The notion of "form" is discussed at some length in the next chapter.

valid arguments, but rather with arguments which are valid in virtue of their form. This being the case, we shall henceforth use the term "valid" to mean *formally valid*, and the term "invalid" to mean not formally valid. Within this terminology, arguments which are not valid at all and arguments which are informally valid will both be counted as "*invalid*."

To be completely precise, we should say that some arguments which are formally valid are valid because of certain *truth-functional* relationships between their premises and conclusion. Such arguments are said to be **truth-functionally valid**, and an argument which is not truth-functionally valid is said to be **truth-functionally invalid**. However another sort of formally valid argument is studied in later chapters, whose validity comes from what are called "quantificational" relationships between premises and conclusion. Such arguments are said to be "quantificationally valid," and an argument which is not quantificationally valid is "quantificationally invalid." For purposes of brevity we shall use the term "valid" to mean "either truth-functionally or quantificationally valid," and we shall use the term "invalid" to mean "either truth-functionally or quantificationally invalid." The term **absolutely invalid** will be reserved for those arguments which are *both* truth-functionally and quantificationally invalid. Thus informally valid arguments will still be regarded as absolutely invalid.*

§8.2   Assessing the Validity of Arguments

Let us now look at a procedure for evaluating the validity or invalidity of truth-functional arguments. This procedure, called the "Truth-Table Method," works because of two facts: (1) an argument is valid if there is no way for all of its premises to be true and its conclusion false; and (2) a properly constructed truth-table shows all the "ways"—the ways in which all the premises can be true—and how each such way must affect the truth-value of the conclusion.

It is conventional in logic to write out arguments as in the example above. First, the premises are written, one below the other. Then the conclusion, prefaced by 'therefore', is written below the premises and separated from them by a line. The argument "If Bill has a new car, then he must be very happy. Bill has a new car. Therefore, he must be very happy." would be written as

> If Bill has a new car then he must be very happy.
> Bill has a new car.
> _____
> Therefore, he must be very happy.

*The reasons for these complications are discussed in Appendix G.

The word 'therefore' is usually abbreviated as a pyramid of three dots ($\therefore$). Thus the above argument, fully abbreviated and translated, would be written out as

$$B \supset H$$
$$B$$
$$\therefore H$$

The truth-table method consists in constructing a truth-table for the argument and then inspecting it to see if it contains the possibility of all premises being true while the conclusion is false. If the table contains such a possibility, the argument is invalid; if it does not, the argument is valid. The specific procedure is this: First, pick out all of the single letters (simple statements) in the argument—premises *and* conclusion; write them above the line, and construct base columns beneath them. Then write each of the premises and the conclusion above the line (the conclusion normally goes to the far right) and, by the procedures already studied, derive columns for each premise and for the conclusion. Next inspect the table row by row to see whether it contains any **invalidating rows**. An invalidating row is one containing a T in the main column for each premise, but an F in the main column for the conclusion. The discovery of such a row will show that it is *possible* for all the premises to be true and the conclusion false simultaneously—if the combination of truth-values in the base columns at that row were to obtain, then the premises would be true and the conclusion false—and hence will show that the argument is invalid.

The truth-table for an argument may contain several invalidating rows. However, even a *single* invalidating row is sufficient to show the argument invalid. If, on the other hand, the truth-table does not contain any invalidating rows, the argument is *valid*. Let us look at an example. The argument:

> If Harvey is going to the party then Clyde is going to the party.
> Clyde isn't going to the party.
> _____
> Therefore, neither Harvey nor Clyde is going to the party.

after suitable abbreviation and translation, becomes

$$H \supset C$$
$$\sim C$$
$$\therefore \sim (H \lor C)$$

The truth-table for this argument is

C	H	$H \supset C$	$\sim C$	$\sim (H \lor C)$
T	T	T	F	F T
T	F	T	F	F T
F	T	F	T	F T
F	F	T	T	T F

In the top row, the conclusion has the value F but one of the premisses also has the value F, so it is not an invalidating row. In the second row, the conclusion has the value F but so has one of the premisses, so it is not an invalidating row. In the third row, the conclusion has the value F but so has one of the premisses, so it is not an invalidating row. In the fourth row, the conclusion has the value T, so it is not an invalidating row. Thus, there are no invalidating rows, which means that the argument is *valid*.

On the other hand, the similar-sounding argument

> If Harvey is going to the party then Clyde is going to the party.
> Harvey isn't going to the party.
> _____
> Therefore, neither Harvey nor Clyde is going to the party.

that is,

$$H \supset C$$
$$\sim H$$
$$\therefore \sim (H \lor C)$$

is shown to be *invalid* by the truth-table

C	H	$H \supset C$	$\sim H$	$\sim (H \lor C)$	
T	T	T	F	F	T
T	F	T	T	F	T ←
F	T	F	F	F	T
F	F	T	T	T	F

whose second row is an invalidating row (indicated by the arrow).

One final illustration emphasizes the fact that, when a truth-table is constructed for an argument, the base columns are contructed for the set of *all* letters in premisses *and* conclusion. Consider the argument:

> If the automobile doesn't start, then something is wrong with the battery.
> Nothing is wrong with the battery.
> _____
> Therefore, if the automobile doesn't start, then something is wrong with the carburetor.

$$\sim A \supset B$$
$$\sim B$$
$$\therefore \sim A \supset C$$

A	B	C	$\sim A \supset B$	$\sim B$	$\sim A \supset C$
T	T	T	F	T	F	F	F	T
T	T	F	F	T	F	F	F	T
T	F	T	F	T	T	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	T	T	F	T	T	T
F	T	F	T	T	F	T	T	F
F	F	T	T	F	T	T	T	T
F	F	F	T	F	T	T	T	F

Is this argument valid or invalid?

Exercises I

Ascertain the validity or invalidity of each of the following arguments.

1. $A \supset B$
   $$\therefore \sim A \supset \sim B$$

2. $A \supset B$
   $$\therefore A \supset (A \cdot B)$$

3. $(A \lor B) \supset (A \cdot B)$
   $$\therefore A \equiv B$$

4. $A \supset (B \cdot C)$
   $\sim B$
   $$\therefore \sim A$$

5. A
   $$\therefore (\sim B \lor B)$$

6. $(A \cdot \sim A)$
   $$\therefore B$$

7. $(A \cdot B)$
   $\sim A$
   $$\therefore C$$

8. $A \supset B$
   $A \supset C$
   $$\therefore A \supset (B \cdot C)$$

9. $A \supset B$
   $A \supset \sim B$
   $$\therefore A$$

10. $A \supset B$
    $\sim A \supset B$
    $$\therefore B$$

11. $A \supset (B \lor C)$
    $B \equiv C$
    $B \supset A$
    $$\therefore \sim C \equiv (\sim B \cdot \sim A)$$

12. $A \supset (B \supset C)$
    $C \supset (D \supset E)$
    $$\therefore A \supset E$$

The truth-table method has two important features. First, it is absolutely *mechanical*, which means that it requires no human judgment and consequently leaves no room for human error, except insofar as humans, not being machines, may make mistakes in applying the mechanical procedure. The truth-table method is such that a machine can do it, and in fact it is a relatively simple procedure to program a computer to evaluate arguments by the truth-table method. Second, the truth-table method is what mathematicians call an **effective** procedure: it has a definite terminating point, and it

is guaranteed to have produced an answer by the time that terminating point is reached. The procedure terminates when the last T or F is placed in the last derived column in the table; when that happens, the resulting truth-table is guaranteed to tell us whether the argument is valid or invalid.

However, from the human standpoint, the truth-table method has one drawback. As the arguments to be evaluated increase in complexity and more and more simple statements are involved, the truth-tables become unendurably long. For example, a complex argument involving twenty simple statements would have something in the neighborhood of a million rows in its truth-table and it is unlikely that even the most persistent logician would have enough patience, or enough pencils, to complete such a diagram. Thus it is desirable to have a less cumbersome method for evaluating complex arguments, even at the expense of mechanical effectiveness. Such a procedure is developed in the next and succeeding chapters.

§8.3 Two Oddities in Truth-Functional Logic

Within truth-functional logic are two rather queer-sounding facts which must be mentioned at some point.* The first is that any argument whose conclusion is a *tautology* is a valid argument, irrespective of the truth-value or logical status of its premises. This is just the reverse of what our intuitions might have led us to expect, since it is tantamount to saying that a statement which is necessarily true (like a tautology) follows from just any old statement you care to name, whereas our intuitions tell us that a necessarily true statement won't follow from anything but another necessarily true statement. This illustrates a point mentioned earlier, that it is unwise to rely too heavily upon intuition when it comes to matters of logic.

The reason why every argument with a tautologous conclusion is valid is this: if a statement is a tautology, then there is no way for it to be false. And if the conclusion to an argument is tautologous, then there is no way for the conclusion of that argument to be false. But clearly, if the conclusion cannot be false *at all*, then it cannot be false when all the premises are true. And this happens to be the defining characteristic of a valid argument: it is valid if its conclusion cannot be false when all of its premises are true. Thus, an argument with a tautologous conclusion is valid because its conclusion cannot be false.

This fact is not *simply* a logical curiosity. It is related, in a fairly complex fashion, to the fact that an argument can be valid even if it has excess premisses—that is, more premises than are needed to establish the conclusion.†

_____
*Sometimes called the "Paradoxes of Entailment" (cf. §13.1), these are related in an obvious way to the so-called "Paradoxes of Material Implication," mentioned in §4.1 and in Appendix E.
†See Appendix H.

And *this* is surely a feature that logic could not do without. A "logic" which maintained that what follows from some information will not follow from that information plus more could not be taken seriously by anyone. Hence we can say that this feature, curious though it may sound, is a necessary ingredient in any satisfactory logical system.

The second logical stepchild is a mirror twin of the first. If one or more statements are such that they cannot all be true simultaneously, they are said to be **incompatible** or **inconsistent**. And any argument whose premises are inconsistent is a valid argument, irrespective of the truth-value or logical status of its conclusion. This too is the opposite of what our intuitions might have led us to expect; intuitively we would probably have said that *nothing at all* follows from inconsistent premises, whereas the fact is that anything and everything follows from inconsistent premises. Still, this principle is not *altogether* contrary to intuition: just as Archimedes said, "Give me a place to stand and I will move the world," so we might say, "Allow me inconsistent premises and I will prove whatever you like." For, starting out with inconsistent premises is only an extreme form of starting out with *false* premises (if they are inconsistent, then at least one of them *must* be false), and surely a person can "prove" anything at all if he is allowed to start out with false premises.

Of course, such an argument (one with inconsistent premises) does not *prove* anything, if by "prove" we mean "establish the truth of." The reason such arguments are valid is also a mirror-image of the reason arguments with tautologous conclusions are valid: if a set of premises is inconsistent (one way for a set of premises to be inconsistent is for one of them to be a contradiction), then they cannot all be true simultaneously. And if an argument is such that it cannot simultaneously have all true premises, then it cannot simultaneously have all true premises and a false conclusion, which is the defining characteristic of a valid argument: it is valid if all its premises cannot be true when its conclusion is false.

These two facts—the two logical "stepchildren"—are related to each other in much the same way that a conditional is related to its contrapositive, which means that they are, in effect, logically equivalent facts. Thus, insofar as we require the first as a necessary ingredient in an adequate logical system, we are forced to take the second along with it. They are not merely twins, they are Siamese twins. But the second fact, unlike the first, may be regarded simply as a logical curio, a conversation-piece of no real utility in its own right, except as it is related to its twin.

Although the intuitive beliefs that a tautology doesn't follow from anything but a tautology, and that nothing follows from a contradiction but another contradiction, are both incorrect, the reverse of each of them is true: nothing follows from a tautology but another tautology, and a contradiction doesn't follow from anything but another contradiction. (Or, to be strictly precise: nothing follows from a necessary truth except another necessary truth, and a necessary falsehood doesn't follow from anything but another

necessary falsehood. This modification is required in order to allow for necessary truths other than tautologies, and necessary falsehoods other than (truth-functional) contradictions.)

§8.4 Showing Truth-Functional Consistency

Although an argument with inconsistent premisses is valid, such an argument *cannot* be sound. A "sound" argument, we may recall, is a valid argument with all true premisses, and if a set of premisses is inconsistent, they cannot all be true; hence an argument with an inconsistent set of premisses, cannot all be true. This shows that validity, though trivially valid, could not possibly be sound. This shows that validity, by itself, is not enough to make an argument a good one. An argument is not a good one unless there is at least a possibility that it does actually establish the truth of its conclusion—that is, unless there is a chance that it is sound. For present purposes, we may define "good argument" as follows:

> *Good Argument.* An argument is a good argument if, and only if, it is a valid argument and has consistent premisses.

Thus, in order to find out whether an argument is a good one we must find out whether it is valid, and also whether its premisses are consistent. As we have already seen, the truth-table provides a way of testing for truth-functional validity. It also provides a way of checking the truth-functional consistency of a set of premisses: if there is any row in the truth-table giving a value of T to *every* premiss, then the set of premisses is **truth-functionally consistent;*** if there is no such row (that is, if each row assigns F to at least one premiss), then the set of premisses is inconsistent, and the argument, though trivially valid, is still a bad argument.

For example, consider the argument '$\sim(A \supset B)$, $\sim(B \supset A) / \therefore \sim B$', and its truth-table:

A	B	$\sim(A \supset B)$		$\sim(B \supset A)$		$\sim B$
T	T	F	T	F	T	F
T	F	T	F	F	T	T
F	T	F	T	T	F	F
F	F	F	T	F	T	T

---

*There are other modes of inconsistency, though truth-functional inconsistency is the only kind revealed by a truth-table. (The two statements "All crows are black" and "Some crows are not black" are incompatible but not truth-functionally so; their incompatibility is *quantificational.*) Thus it is prudent to exercise some caution in characterizing arguments as good or bad on the basis of truth-tables. If an argument is shown valid by a truth-table, and has truth-functionally consistent premisses, then we can say that *truth-function-ally* it is a good argument; but there is still the possibility that it may be a bad argument because of some unrevealed kind of incompatibility among its premisses.

Lines 1 and 3 are the ones where the conclusion is false; but at line 1 both premisses are also false, and at line 3 the first premiss is also false. Thus, there are no invalidating lines, and so the argument is valid. However, at each line at least one of the premisses is false. Therefore, the premisses are inconsistent, and so the argument, though valid, is no good. On the other hand, consider the argument '$A \supset B$, $C \supset \sim B / \therefore \sim(A \cdot C)$' and its truth-table:

	A	B	C	$A \supset B$	$C \supset \sim B$		$\sim(A \cdot C)$	
(1)	T	T	T	T	F	F	F	T
(2)	T	T	F	T	T	F	T	F
(3)	T	F	T	F	T	T	F	T
(4)	T	F	F	F	T	T	T	F
(5)	F	T	T	T	F	F	T	F
(6)	F	T	F	T	T	F	T	F
(7)	F	F	T	T	T	T	T	F
(8)	F	F	F	T	T	T	T	F

Lines 1 and 3 are the ones where the conclusion is false; but at line 1 the second premiss is false and at line 3 the first premiss is false. Thus this argument is also valid. But at line 2 (and also lines 6, 7, and 8) both premisses are true. This shows that it is possible for the premisses all (both) to be true—that they are consistent. Therefore, this argument is a good one.

Exercises II

Decide which of the arguments in Exercises I are good arguments and which are not.